

TEST #2 @ 200 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. For an exercise to be complete there needs to be a detailed solution to the problem. No proof, no credit given! Clearly label each exercise.

1. Graph the solution set of the linear inequality $2x + 3y > -6$. Show all work, and label all the points.

2. Simplify each expression. Write the final answer using only positive exponents (if necessary).

a) $\left(\frac{3x^{-1}p}{x^5p^4}\right)^{-2}$

b) $2^{-2} + \left(\frac{1}{3}\right)^{-1} + (-2)^{-1} + 3^0$

3. Perform the following division using long division: $\frac{4x^3 - 4x^2 + 5x - 8}{2x - 1}$

4. Factor each expression as completely as possible. If prime, state so.

a) $4m^5 + 12m^4 - 40m^3$

e) $6r^2 + r - 1$

h) $1 - 81p^4$

b) $8p^2 + 14p + 5$

f) $2x + x^2 - 15$

i) $25x^2 - 1$

c) $a^2 - 4a - ab + 4b$

g) $3x^2 - 11x - 20$

j) $6a^2 + 40a + 24$

d) $27y^3 - 8$

5. Do the following operations and simplify:

a) $(2a + 3b)^2$

b) $(5\sqrt{7} - 2\sqrt{3})^2$

c) $(5 + \sqrt{3x-1})^2$

d) $(5\sqrt{7} - 2\sqrt{3})(5\sqrt{7} + 2\sqrt{3})$

e) $\left(5p - \frac{1}{2}\right)\left(5p + \frac{1}{2}\right)$

f) $4\sqrt{50} + 3\sqrt{12} - 5\sqrt{45}$

g) $(x-1)^3$

h) $\frac{16 + 8\sqrt{2}}{24}$

i) $\frac{x}{x+2} + \frac{5-x}{x} - \frac{4}{x^2+2x}$

j) $\frac{m^2 - 4}{m^2 - 1} \div \frac{2m^2 + 4m}{1 - m}$

k) $\frac{1+t^3}{1+t}$

l) $2\sqrt[3]{32m^3} - \sqrt[3]{108m^3}$

m) $\frac{x+5}{x+10} \div \left(\frac{x^2+10x+25}{x^2+10x} \cdot \frac{10x}{x^2+15x+50}\right)$

$$\begin{array}{r} 2 \overline{) 108} \\ \underline{4} \\ 64 \\ \underline{64} \\ 0 \end{array}$$

6. Solve the following equations by factoring.

a) $y^2 + 2y = 0$

d) $x^3 + 4x^2 + 3x = 0$

b) $(3x - 5)(4x + 1) = 24$

e) $9y^3 = 49y$

c) $3x^2 - 21x = -30$

f) $5x\left(x - \frac{3}{2}\right)\left(5x - \frac{2}{3}\right)\left(3 - \frac{x}{4}\right) = 0$

7. Solve the following equations. Write conditions or check your solutions, whichever is appropriate.

a) $\frac{1}{x-1} + \frac{1}{2} = \frac{2}{x^2-1}$

b) $\frac{x}{4-x} = \frac{2}{x}$

c) $\frac{3}{x+3} + \frac{4}{x+6} = \frac{9}{x^2+9x+18}$

8) Rationalize each denominator and simplify.

a) $\frac{2}{\sqrt{3}}$

b) $\frac{5}{\sqrt[3]{2}}$

c) $\frac{1}{4+\sqrt{15}}$

9. An object projected from a height with an initial velocity of 32 ft per sec after t seconds has height

$$h = -16t^2 + 32t + 48$$

- a) What is the initial height of the object?
 - b) What is the height of the object after 1 second?
 - c) After how many seconds is the height 64 ft?
 - d) After how many seconds does the object hit the ground?
-

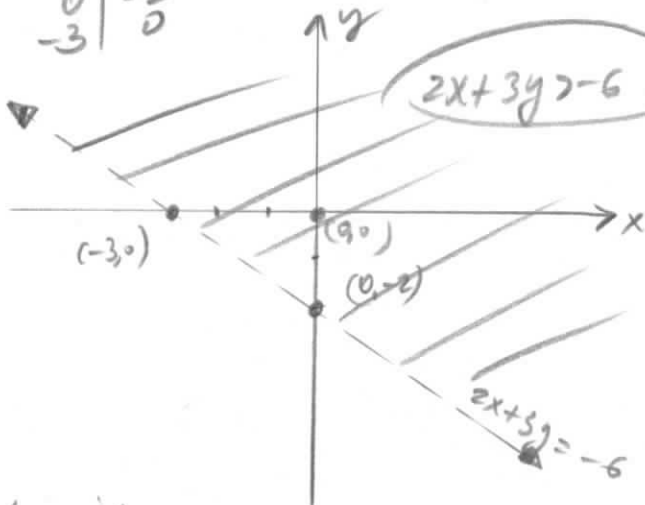
10. Mark can tune up his car in 2 hours working alone. Ben can do the job in 3 hours working alone. How long would it take them if they worked together?

11. Vince can fly his plane 200 mi against the wind in the same time it takes him to fly 300 mi with the wind. Find the speed of his plane in still air if the wind blows at 30 mph.

$$\textcircled{1} \quad 2x + 3y > -6$$

Boundary line: $2x + 3y = -6$
(dashed)

x	y
0	-2
-3	0



Test point

let $(0,0)$ & line

$$2(0) + 3(0) > -6$$

$$0 > -6 \text{ TRUE}$$

$\Rightarrow (0,0) = \text{solution}$

$$\begin{aligned} \textcircled{2} \text{ (a)} \quad \left(\frac{3x^{-1}p}{x^5p^4} \right)^{-2} &= \left(3x^{-1-5} p^{1-4} \right) \\ &= \left(3x^{-6} p^{-3} \right)^{-2} = 3^{-2} (x^{-6})^{-2} (p^{-3})^{-2} \\ &= \frac{1}{9} x^{12} p^6 = \boxed{\frac{x^{12} p^6}{9}} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad 2^{-2} + \left(\frac{1}{3}\right)^{-1} + (-2)^{-1} + 3^0 &= \\ &= \frac{1}{2^2} + \frac{1}{\frac{1}{3}} + \frac{1}{-2} + 1 \\ &= \frac{1}{4} + \frac{3}{1} - \frac{1}{2} + \frac{1}{1} = \frac{-1}{4} + \frac{4}{1} \\ &= \frac{-1+16}{4} = \boxed{\frac{15}{4}} \end{aligned}$$

$$\begin{array}{r} 2x^2 - \cancel{4x} + 2 \\ 2x-1 \overline{) 4x^3 - 4x^2 + 5x - 8} \\ \underline{-4x^3 + 2x^2} \\ -2x^2 + 5x - 8 \\ \underline{+ 2x^2 - 2x} \\ 3x - 8 \\ \underline{+ 2x^2 - 2x} \\ x - 8 \\ \underline{-4x + 2} \\ -6 \end{array}$$

$$\frac{4x^3 - 4x^2 + 5x - 8}{2x-1} = \left[2x^2 - x + 2 - \frac{6}{2x-1} \right]$$

$$\begin{aligned} \textcircled{4} \text{ (c)} \quad 4m^5 + 12m^4 - 40m^3 &= \\ \text{GCF} &= 4m^3 \\ &= 4m^3(m^2 + 3m - 10) \\ \text{product} &= c = -10 < \begin{matrix} +5 \\ -2 \end{matrix} \\ \text{sum} &= b = 3 \\ \hline &= 5 \cdot 2 \\ &= 10 = 5 \cdot 2 \\ &= \boxed{4m^3(m+5)(m-2)} \end{aligned}$$

(b)

$$\begin{array}{l} 8p^2 + 14p + 5 \\ a+1 \left(\begin{array}{l} \text{Split middle term} \\ \text{product} = ac = 8 \cdot 5 = 40 < \begin{matrix} +4 \\ +10 \end{matrix} \\ \text{sum} = b = 14 \\ \hline 40 = 4 \cdot 10 \end{array} \right) \end{array}$$

$$\begin{aligned} 8p^2 + 14p + 5 &= 8p^2 + 4p + 10p + 5 \\ &= 4p(2p+1) + 5(2p+1) \\ &= \boxed{(2p+1)(4p+5)} \end{aligned}$$

c) $a^2 - 4a - ab + 4b =$
 $= a(a-4) - b(a-4)$
 $= (a-4)(a-b)$

d) $27y^3 - 8 = (3y)^3 - 2^3$
 $= (3y-2)((3y)^2 + 3y \cdot 2 + 2^2)$
 $= (3y-2)(9y^2 + 6y + 4)$

e) $6r^2 + r - 1 = ?$
 $a \neq 1$, split the middle term
 product = $ac = 6(-1) = -6$
 sum = $b = 1$
 $6 = 3 \cdot 2$

$? = 6r^2 + 3r - 2r - 1$
 $= 3r(2r+1) - (2r+1)$
 $= (2r+1)(3r-1)$

f) $2x + x^2 - 15 =$
 $x^2 + 2x - 15 = (x+5)(x-3)$
 $a=1$, product = $c = -15$
 sum = $b = 2$
 $15 = 5 \cdot 3$

g) $3x^2 - 11x - 20 = ?$
 $a \neq 1$, split the middle term
 product = $ac = 3(-20) = -60$
 sum = -11
 $60 = 15 \cdot 4$

$? = 3x^2 + 4x - 15x - 20$
 $= x(3x+4) - 5(3x+4)$
 $= (3x+4)(x-5)$

h) $1 - 81p^4 = 1^2 - (9p^2)^2$
 difference of squares
 $= (1-9p^2)(1+9p^2)$
 $= (1^2 - (3p)^2)(1+9p^2)$
 diff. of squares & prime
 $= (1-3p)(1+3p)(1+9p^2)$

i) $25x^2 - 1 = (5x)^2 - 1^2$
 difference of squares
 $= (5x-1)(5x+1)$

j) $6a^2 + 40a + 24 =$
 GCF = 2
 $= 2(3a^2 + 20a + 12)$
 split middle term
 product = $3 \cdot 12 = 36$
 sum = 20
 $36 = 18 \cdot 2$
 $= 2(3a^2 + 18a + 2a + 12)$
 $= 2[3a(a+6) + 2(a+6)]$
 $= 2(a+6)(3a+2)$

$$(5) (a) (2a+3b)^2 = (2a)^2 + 2(2a)(3b) + (3b)^2 = 4a^2 + 12ab + 9b^2$$

$$(b) (5\sqrt{7} - 2\sqrt{3})^2 = (5\sqrt{7})^2 - 2(5\sqrt{7})(2\sqrt{3}) + (2\sqrt{3})^2 = 25 \cdot 7 - 20\sqrt{21} + 4 \cdot 3 = 187 - 20\sqrt{21}$$

$$(c) (5 + \sqrt{3x-1})^2 = 5^2 + 2 \cdot 5 \sqrt{3x-1} + (\sqrt{3x-1})^2 = 25 + 10\sqrt{3x-1} + 3x - 1 = 24 + 10\sqrt{3x-1} + 3x$$

$$(d) (5\sqrt{7} - 2\sqrt{3})(5\sqrt{7} + 2\sqrt{3}) = \text{diff. of squares} = (5\sqrt{7})^2 - (2\sqrt{3})^2 = 25 \cdot 7 - 4 \cdot 3 = 163$$

$$(e) (5p - \frac{1}{2})(5p + \frac{1}{2}) = \text{diff. of squares} = (5p)^2 - (\frac{1}{2})^2 = 25p^2 - \frac{1}{4}$$

$$(f) 4\sqrt{50} + 3\sqrt{12} - 5\sqrt{45} = 4\sqrt{25 \cdot 2} + 3\sqrt{4 \cdot 3} - 5\sqrt{9 \cdot 5} = 4 \cdot 5\sqrt{2} + 3 \cdot 2\sqrt{3} - 5 \cdot 3\sqrt{5} = 20\sqrt{2} + 6\sqrt{3} - 15\sqrt{5}$$

$$(g) (x-1)^3 = (x-1)^2(x-1) = (x^2 - 2x + 1)(x-1) = x^3 - x^2 - 2x^2 + 2x + x - 1 = x^3 - 3x^2 + 3x - 1$$

$$(h) \frac{16 + 8\sqrt{2}}{24} = \frac{8(2 + \sqrt{2})}{24} = \frac{2 + \sqrt{2}}{3}$$

$$(i) \frac{x}{x+2} + \frac{5-x}{x} - \frac{4}{x^2+2x} = \frac{x}{x+2} + \frac{5-x}{x} - \frac{4}{x(x+2)}$$

LCD = $x(x+2)$

$$= \frac{x^2 + (x+2)(5-x) - 4}{x(x+2)} = \frac{x^2 + 5x - x^2 + 10 - 2x - 4}{x(x+2)} = \frac{3x + 6}{x(x+2)} = \frac{3(x+2)}{x(x+2)} = \frac{3}{x}$$

$$(j) \frac{m^2 - 4}{m^2 - 1} \div \frac{2m^2 + 4m}{1 - m} =$$

$$= \frac{(m-2)(m+2)}{(m-1)(m+1)} \cdot \frac{1-m}{2m(m+2)} = \frac{(m-2)(-1)(m-1)}{(m-1)(m+1)2m} = \frac{-(m-2)}{2m(m+1)} = \frac{2-m}{2m(m+1)}$$

-4-

$$\begin{aligned}
 (k) \quad \frac{1+t^3}{1+t} &= \frac{1+t^3}{1+t} \\
 &= \frac{(1+t)(1-t+t^2)}{1+t} \\
 &= \boxed{1-t+t^2}
 \end{aligned}$$

$$\begin{aligned}
 (l) \quad 2\sqrt[3]{32m^3} - \sqrt[3]{108m^3} &= \\
 &= 2\sqrt[3]{8 \cdot 4m^3} - \sqrt[3]{27 \cdot 4} \\
 &= 2 \cdot 2m\sqrt[3]{4} - 3\sqrt[3]{4} \\
 &= \boxed{4m\sqrt[3]{4} - 3\sqrt[3]{4}}
 \end{aligned}$$

$$\begin{aligned}
 (m) \quad \frac{x+5}{x+10} &\div \left(\frac{x^2+10x+25}{x^2+10x} \cdot \frac{10x}{x^2+15x+5} \right) \\
 &= \frac{x+5}{x+10} \div \left(\frac{(x+5)^2}{x(x+10)} \cdot \frac{10x}{(x+10)(x+5)} \right) \\
 &= \frac{x+5}{x+10} \div \frac{10(x+5)}{(x+10)^2} \\
 &= \frac{x+5}{x+10} \cdot \frac{(x+10)^2}{10(x+5)} = \boxed{\frac{x+10}{10}}
 \end{aligned}$$

$$\begin{aligned}
 (6) (a) \quad y^2 + 2y &= 0 \\
 y(y+2) &= 0 \\
 \text{Zero-Factor property} &\Rightarrow \\
 y=0 \quad \text{or} \quad y+2=0 \\
 &\quad \quad \quad y=-2 \\
 &= \boxed{y \in \{0, -2\}}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad (3x-5)(4x+1) &= 24 \\
 12x^2 + 3x - 20x - 5 - 24 &= 0 \\
 12x^2 - 17x - 29 &= 0 \\
 a \neq 1, \text{ we'll split middle term} & \\
 \text{product} = ac = 12(-29) &< \begin{matrix} +12 \\ -29 \end{matrix} \\
 \text{sum} = b = -17 & \\
 \hline &12 \cdot 29
 \end{aligned}$$

$$\begin{aligned}
 12x^2 + 12x - 29x - 29 &= 0 \\
 12x(x+1) - 29(x+1) &= 0 \\
 (x+1)(12x-29) &= 0 \\
 \text{Zero-factor prop.} &\Rightarrow \\
 x+1=0 \quad \text{or} \quad 12x-29=0 \\
 x=-1 &\quad \quad \quad x = \frac{29}{12}
 \end{aligned}$$

$$\boxed{x \in \left\{ -1, \frac{29}{12} \right\}}$$

$$\begin{aligned}
 (c) \quad 3x^2 - 21x &= -30 \\
 3x^2 - 21x + 30 &= 0 \quad \Big/ :3 \\
 x^2 - 7x + 10 &= 0 \\
 (x-2)(x-5) &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{Zero-factor prop.} &\Rightarrow \\
 x-2=0 \quad \text{or} \quad x-5=0 \\
 x=2 &\quad \quad \quad x=5
 \end{aligned}$$

$$\boxed{x \in \{2, 5\}}$$

(d) $x^3 + 4x^2 + 3x = 0$
 $x(x^2 + 4x + 3) = 0$
 $x(x+3)(x+1) = 0$
 Zero-factor prop. \Rightarrow
 $x=0$ OR $x+3=0$ OR $x+1=0$
 $x=-3$ $x=-1$

$$x \in \{0, -3, -1\}$$

(e) $9y^3 = 49y$
 $9y^3 - 49y = 0$
 GCF = y

$$y(9y^2 - 49) = 0$$

$$y((3y)^2 - 7^2) = 0$$

$$y(3y-7)(3y+7) = 0$$

Zero-factor prop. \Rightarrow

$$y=0 \text{ OR } 3y-7=0 \Rightarrow y=\frac{7}{3}$$

$$\text{OR } 3y+7=0 \Rightarrow y=-\frac{7}{3}$$

$$y \in \left\{0, \frac{7}{3}, -\frac{7}{3}\right\}$$

(f) $5x(x-\frac{3}{2})(5x-\frac{2}{3})(3-\frac{x}{4}) = 0$
 Zero-Factor Property \Rightarrow

$$x=0 \text{ OR}$$

$$x-\frac{3}{2}=0 \Rightarrow x=\frac{3}{2} \text{ OR}$$

$$5x-\frac{2}{3}=0 \Rightarrow 5x=\frac{2}{3} \Rightarrow x=\frac{2}{3} \cdot \frac{1}{5} = \frac{2}{15}$$

$$3-\frac{x}{4}=0 \Rightarrow 3=\frac{x}{4} \Rightarrow x=12$$

$$x \in \left\{0, \frac{3}{2}, \frac{2}{15}, 12\right\}$$

(7) (a) $\frac{1}{x-1} + \frac{1}{2} = \frac{2}{x^2-1}$
 $\frac{2(x+1)}{2(x+1)} \cdot \frac{1}{x-1} + \frac{(x+1)(x-1)}{(x+1)(x-1)} \cdot \frac{2}{2} = \frac{2}{(x-1)(x+1)}$ / 2(x+1)

conditions $\begin{cases} x-1 \neq 0 \\ x+1 \neq 0 \end{cases} \Rightarrow x \neq \pm 1$

$$2(x+1) + (x+1)(x-1) = 4$$

$$2(x+1) + (x+1)(x-1) = 4$$

$$2x+2 + x^2-1 = 4$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0 \Rightarrow$$

$$x+3=0 \text{ OR } x-1=0$$

$$x=-3$$

$$x=1$$

not possible

$$x \in \{-3\}$$

(b) $\frac{x}{4-x} = \frac{2}{x}$

conditions

$$x \neq 4$$

$$x \neq 0$$

Cross-product

property \Rightarrow

$$x^2 = 2(4-x)$$

$$x^2 = 8 - 2x$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x=-4, \quad x=2$$

$$x \in \{-4, 2\}$$

$$(c) \frac{3}{x+3} + \frac{4}{x+6} = \frac{9}{x^2+9x+18}$$

$$\frac{3}{x+3} + \frac{4}{x+6} = \frac{9}{(x+3)(x+6)}$$

$$LCD = (x+3)(x+6)$$

Conditions: $x \neq -3, x \neq -6$

$$3(x+6) + 4(x+3) = 9$$

$$3x + 18 + 4x + 12 = 9$$

(linear equation)

$$7x + 30 = 9$$

$$7x = 9 - 30$$

$$7x = -21$$

$x = -3$ not possible

$$x \in \emptyset \quad (\text{No solutions})$$

$$(P)(a) \frac{\frac{13}{2}}{\sqrt{3}} = \frac{2\sqrt{3}}{(\sqrt{3})^2} = \boxed{\frac{2\sqrt{3}}{3}}$$

$$(b) \frac{\sqrt[3]{9}}{5} = \frac{5\sqrt[3]{4}}{\sqrt[3]{2} \sqrt[3]{2} \sqrt[3]{4}}$$

$$= \frac{5\sqrt[3]{4}}{\sqrt[3]{8}} = \boxed{\frac{5\sqrt[3]{4}}{2}}$$

$$(c) \frac{4-\sqrt{15}}{1} = \frac{4-\sqrt{15}}{(4+\sqrt{15})(4-\sqrt{15})}$$

$$= \frac{4-\sqrt{15}}{4^2 - (\sqrt{15})^2} = \frac{4-\sqrt{15}}{16-15}$$

$$= \boxed{4-\sqrt{15}}$$

$$(9) h = -16t^2 + 32t + 48$$

$t = \text{time (seconds)}$
 $h = \text{height (feet)}$

(a) find h when $t=0$
 if $t=0$, then $h = 48$ ft
 The initial height of the object was 48 ft.

(b) find h when $t=1$
 if $t=1$, then $h = -16 + 32 + 48$
 $h = 64$ ft
 After 1 second, the object is at 64 ft.

(c) find t when $h = 64$ ft
 solve: $64 = -16t^2 + 32t + 48$
 (quadratic equation)
 solve by factoring:

$$16t^2 - 32t - 48 + 64 = 0$$

$$16t^2 - 32t + 16 = 0 \quad \div: 16$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0 \Rightarrow t-1 = 0$$

$t = 1$ second

The object is at 64 ft only after 1 second (this is the maximum height)

(d) find t when $h = 0$
 if $h = 0$, solve $-16t^2 + 32t + 48 = 0$
 solve by factoring:
 $16t^2 - 32t - 48 = 0 \quad \div: 16$
 $t^2 - 2t - 3 = 0$

(10) Given:

	time needed to finish the job	part of job done in hours
Mark	2 hr	$\frac{1}{2}$
Ben	3 hr	$\frac{1}{3}$
together	x hr	$\frac{1}{x}$

Find: How many hours to finish the job together

Let x = the number of hours needed to finish the job together

Then, $\frac{1}{2} + \frac{1}{3} = \frac{1}{x}$

LCO = 6

$$\frac{3+2}{6} = \frac{1}{x}$$

$$\frac{5}{6} = \frac{1}{x}$$

(cross-product property =)

$$5x = 6$$

$$x = \frac{6}{5} = 1\frac{1}{5} \text{ h}$$

It takes them $1\frac{1}{5}$ hour (1 hour and 12 minutes) to finish the job together

(11) Given

	distance	rate	time
against wind	200 mi	$x-30$ mph	same time
with wind	300 mi	$x+30$ mph	

Find the speed of plane in still air
let x = speed of plane in still air

$$\text{time against wind} = \text{time with wind}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\frac{200}{x-30} = \frac{300}{x+30} \quad \text{Note } \begin{pmatrix} x \neq 30 \\ x \neq -30 \end{pmatrix}$$

Divide both sides by 100 =

$$\frac{2}{x-30} = \frac{3}{x+30}$$

cross-product property:

$$2(x+30) = 3(x-30)$$

$$2x+60 = 3x-90$$

(linear equation)

$$60+90 = 3x-2x$$

$$150 = x$$

The speed of the plane in still air was 150 mph.