Math 51 Summer 2011 Name:

QUIZ #2 @ 100 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Factor each expression as completely as possible. If prime, state so.

a) $2x-6-ax+3a$

b) $3(5-x)+y(5-x)$

c) $m^2+9m+14$

d) $36-x^2$

e) $y^2-5y+12$

f) $2x^6+8x^5-42x^4$

g) $d^2+4d-45$

h) $m^2-13m-30$

i) x^4-16

j) $3t^2-11t-20$

k) $3y^2-48y+192$

l) $2x^2-10x+3xy-15y$

m) a^2+16

n) $8x^3-1$

o) $54+2y^3$

---

2. . Solve the following equations by factoring.

a) $x(2x-3)=-1$

b) $15z^2=15z$

c) $(a-2)(a-4)=15$

d) $10y^2+43y=9$

e) $5-(x-1)^2=(x-2)^2$

f) $t^3+35t=12t^2$

g) $x^2-6x-7=0$

---

3. The length of a rectangular flower bed is 3 ft less than twice its width. The area of the bed is 54 sq.ft. Find the dimensions of the flower bed. (Clearly state the given, what needs to be found, and the variable(s).)

MATH 51

Quiz 2

(1) (a) $2x - 6 - ax + 3a =$

$2(x-3) - a(x-3) =$

$\boxed{(x-3)(2-a)}$

(b) $3(5-x) + y(5-x) =$

$\boxed{(5-x)(3+y)}$

(c) $m^2 + 9m + 14 = \boxed{(m+7)(m+2)}$

product $= c = 14 < {}^{+7}_{+2}$

sum $= b = 9$

$14 = 7 \cdot 2$

(d) $36 - x^2 = 6^2 - x^2$ diff. of squares

$= \boxed{(6-x)(6+x)}$

(e) $y^2 - 5y + 12 = \boxed{\text{prime}}$

product $= c = 12 < {}^{-}_{-}$

sum $= b = -5$

$12 = 6 \cdot 2 = 4 \cdot 3 = 12 \cdot 1$

not possible to find two factors whose sum is -5

OR check $b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 12$

$= 25 - 48 < 0$

not a perfect square, so the trinomial is not factorable

(f) $2x^6 + 8x^5 - 42x^4 =$

GCF $= 2x^4$

$= 2x^4(x^2 + 4x - 21)$

product $= c = -21 < {}^{+7}_{-3}$

sum $= b = 4$

$21 = 7 \cdot 3$

$= \boxed{2x^4(x+7)(x-3)}$

(g) $d^2 + 4d - 45 = \boxed{(d+9)(d-5)}$

product $= c = -45 < {}^{+9}_{-5}$

sum $= b = 4$

$45 = 9 \cdot 5$

(h) $m^2 - 13m - 30 = \boxed{(m+2)(m-15)}$

product $= c = -30 < {}^{+2}_{-15}$

sum $= b = -13$

$30 = 3 \cdot 10 = 6 \cdot 5 = 15 \cdot 2$

(i) $x^4 - 16 = (x^2)^2 - 4^2$ diff. of sq.

$= (x^2 - 4)(x^2 + 4)$

$= (x^2 - 2^2)(x^2 + 4)$ diff. of sq.

$= \boxed{(x-2)(x+2)(x^2+4)}$

(j) $3t^2 - 11t - 20 = \%$

product $= ac = 3(-20) = -60 < {}^{+4}_{-15}$

sum $= b = -11$

$60 = 15 \cdot 4$

$\% = 3t^2 + 4t - 15t - 20$

$= t(3t+4) - 5(3t+4)$

$= \boxed{(3t+4)(t-5)}$

(k) $3y^2 - 48y + 192 =$

GCF $= 3$

$= 3(y^2 - 16y + 64)$ perfect square trinomial

$= \boxed{3(y-8)^2}$

(l) $2x^2 - 10x + 3xy - 15y =$

$2x(x-5) + 3y(x-5) =$

$\boxed{(x-5)(2x+3y)}$

(m) $a^2 + 16 = \boxed{\text{prime}}$

(n) $8x^3 - 1 = (2x)^3 - 1^3$

diff. of cubes

$= (2x-1)((2x)^2 + 2x \cdot 1 + 1^2)$

$= \boxed{(2x-1)(4x^2 + 2x + 1)}$

(o) $54 + 2y^3 =$

GCF = 2

$= 2(27 + y^3)$

$= 2(3^3 + y^3)$

sum of cubes

$= 2(3+y)(3^2 - 3y + y^2)$

$= \boxed{2(3+y)(9 - 3y + y^2)}$

(2) (a) $x(2x-3) = -1$

$2x^2 - 3x = -1$

quadratic eq.

we'll solve it by factoring:

$2x^2 - 3x + 1 = 0$

product = ac = 2 < -1, -2

sum = b = -3

$2 = 1 \cdot 2$

$\underbrace{2x^2 - x}\ \underbrace{- 2x + 1} = 0$

$x(2x-1) - (2x-1) = 0$

$(2x-1)(x-1) = 0$

By zero-factor property $\Rightarrow$

$2x - 1 = 0$ OR $x - 1 = 0$

$2x = 1$ $\quad x = 1$

$x = \frac{1}{2}$

$\boxed{x \in \{\tfrac{1}{2}, 1\}}$

(b) $15z^2 = 15z$ quadratic eq.

$15z^2 - 15z = 0$ GCF = $15z$

$15z(z-1) = 0$

zero-factor prop. $\Rightarrow$

$z = 0$ OR $z - 1 = 0$

$z = 1$

$\boxed{z \in \{0, 1\}}$

(c) $(a-2)(a-4) = 15$

$a^2 - 4a - 2a + 8 = 15$

quadratic eq.

$a^2 - 6a + 8 - 15 = 0$

$a^2 - 6a - 7 = 0$

product = c = -7 < $+1$, -7

sum = b = -6

$7 = 7 \cdot 1$

$(a+1)(a-7) = 0$

zero-factor prop. $\Rightarrow$

$a + 1 = 0$ OR $a - 7 = 0$

$a = -1$ $\quad a = 7$

$\boxed{a \in \{-1, 7\}}$

(d) $10y^2 + 43y = 9$

quadratic eq.

$10y^2 + 43y - 9 = 0$

$\left[\begin{array}{l} \text{product} = ac = 10(-9) = -90 < {}^{+45}_{-2} \\ \text{sum} = b = 43 \\ \hline 90 = 45 \cdot 2 \end{array}\right.$

$\underbrace{10y^2 + 45y} \underbrace{- 2y - 9} = 0$

$5y(2y+9) - (2y+9) = 0$

$(2y+9)(5y-1) = 0$

zero-factor property $\Rightarrow$

$2y + 9 = 0$ OR $5y - 1 = 0$

$2y = -9$ $\quad$ $5y = 1$

$y = \frac{-9}{2}$ $\quad$ $y = \frac{1}{5}$

$\boxed{y \in \{-\frac{9}{2}, \frac{1}{5}\}}$

(e) $5 - (x-1)^2 = (x-2)^2$

$5 - (x^2 - 2x + 1) = x^2 - 4x + 4$

$5 - x^2 + 2x - 1 = x^2 - 4x + 4$

quadratic eq

$0 = x^2 - 4x + \cancel{4} - \cancel{4} + x^2 - 2x$

$2x^2 - 6x = 0$ $\quad$ GCF $= 2x$

$2x(x-3) = 0$

zero-factor prop. $\Rightarrow$

$x = 0$ OR $x - 3 = 0$

$x = 3$

$\boxed{x \in \{0, 3\}}$

(f) $t^3 + 35t = 12t^2$

third-degree eq.

We'll solve it by factoring

$t^3 - 12t^2 + 35t = 0$

GCF $= t$

$t(t^2 - 12t + 35) = 0$

$\left[\begin{array}{l} \text{product} = c = 35 < {}^{-7}_{-5} \\ \text{sum} = b = -12 \\ \hline 35 = 7 \cdot 5 \end{array}\right.$

$t(t-7)(t-5) = 0$

zero-factor property $\Rightarrow$

$t = 0$ OR $t - 7 = 0$ OR $t - 5 = 0$

$t = 7$ $\quad$ $t = 5$

$\boxed{t \in \{0, 7, 5\}}$

(g) $x^2 - 6x - 7 = 0$

$\left[\begin{array}{l} \text{quadratic eq.} \\ \text{product} = c = -7 < {}^{+1}_{-7} \\ \text{sum} = b = -6 \\ \hline 7 = 7 \cdot 1 \end{array}\right.$

$(x+1)(x-7) = 0$

zero-factor prop. $\Rightarrow$

$x + 1 = 0$ OR $x - 7 = 0$

$x = -1$ $\quad$ $x = 7$

$\boxed{x \in \{-1, 7\}}$

(3) Given:

- rectangle
- length is 3 ft less than twice width
- area is 54 ft^2

Find

- width and length

Solution

let w = width (in ft)

then $2w - 3$ = length

Area = width · length

$w(2w-3) = 54$

$2w^2 - 3w = 54$

(quadratic eq)

We'll solve it by the factoring method

$2w^2 - 3w - 54 = 0$

$a \neq 1$, so we'll split the middle term

product $= ac = 2(-54)$ $\begin{cases} +9 \\ -12 \end{cases}$

sum $= b = -3$

$2 \cdot 54 = 108$

$108 = \cancel{4 \cdot 27}$

$= 12 \cdot 9$

$2w^2 + 9w - 12w - 54 = 0$

$w(2w+9) - 6(2w+9) = 0$

$(2w+9)(w-6) = 0$

zero factor property =>

$2w+9=0$ OR $w-6=0$

$2w = -9$ $\qquad w = 6$

$w = -\frac{9}{2}$

not positive

width is 6 ft and length is $2w - 3 = 9$ ft