

Quiz 2 - Solutions

(1) (a) $a^{-2} + 2a^{-2} = 3a^{-2} = \boxed{\frac{3}{a^2}}$

(b) $x^2 - 2x^3$ - cannot be simplified (not like terms).

(c) $t^{-1} - t^{-1} = \boxed{0}$

(d) $\frac{(2x^{-1}y)^2}{x^{-3}y^5} = \frac{2^2(x^{-1})^2y^2}{x^{-3}y^5}$
 $= \frac{4x^{-2}y^2}{x^{-3}y^5} = 4x^{-2-(-3)}y^{2-5}$
 $= 4x^{2+3}y^{-3} = \boxed{\frac{4x}{y^3}}$

(e) $3^{-2} + \left(\frac{1}{2}\right)^{-1} + (-2)^{-1} + 5^0 =$

$\frac{1}{3^2} + \frac{1}{\frac{1}{2}} + \frac{1}{-2} + 1 =$

$\frac{1}{9} + \frac{2}{1} - \frac{1}{2} + \frac{1}{1} = \frac{2}{9} - \frac{1}{2} + \frac{18}{9}$

$LCM = 18$

$= \frac{2}{18} - \frac{9}{18} + \frac{54}{18} = \boxed{\frac{47}{18}}$

(f) $(y^3 - 5)^2 = (y^3)^2 - 2 \cdot y^3 \cdot 5 + 5^2$
 $= \boxed{y^6 - 10y^3 + 25}$

(g) $(9y + \frac{2}{3})(9y - \frac{2}{3}) =$
 $(9y)^2 - \left(\frac{2}{3}\right)^2 = \boxed{81y^2 - \frac{4}{9}}$

(h) $2x(5x-1)(5x+1) =$

$2x((5x)^2 - 1^2) =$

$2x(25x^2 - 1) =$

$\boxed{50x^3 - 2x}$

(i) $(t^{-1})(-t^{-1}) = -t^{-1-1} = -t^{-2}$
 $= \boxed{\frac{-1}{t^2}}$

(j) $\left(\frac{1}{3}a - c\right)^2 =$

$= \left(\frac{1}{3}a\right)^2 - 2\left(\frac{1}{3}a\right) \cdot c + c^2$

$= \boxed{\frac{1}{9}a^2 - \frac{2}{3}ac + c^2}$

(2) $\frac{x^4 - 4x^3 + 5x^2 - 3x + 2}{x^2 + 3} =$

$$\begin{array}{r} x^2 + 3 \overline{) x^4 - 4x^3 + 5x^2 - 3x + 2} \\ \underline{-x^4 + 3x^2} \\ 1 - 4x^3 + 2x^2 - 3x + 2 \\ \underline{+ 4x^3 + 12x} \\ 1 2x^2 + 9x + 2 \\ \underline{- 2x^2 - 6} \\ 9x - 4 \end{array}$$

So,

$\frac{x^4 - 4x^3 + 5x^2 - 3x + 2}{x^2 + 3} = x^2 - 4x + \frac{9x - 4}{x^2 + 3}$

-2-

$$(3) (a) \quad 2x - 6 - ax + 3a =$$

$$2(x-3) - a(x-3) =$$

$$\boxed{(x-3)(2-a)}$$

$$(b) \quad 3(5-x) + y(5-x) =$$

$$\boxed{(5-x)(3+y)}$$

$$(c) \quad m^2 + 9m + 14 = \boxed{(m+7)(m+2)}$$

product = 14 < $\begin{matrix} +7 \\ +2 \end{matrix}$
 sum = 9
 $14 = 7 \cdot 2$

$$(d) \quad 36 - x^2 = 6^2 - x^2 \text{ difference of squares}$$

$$= \boxed{(6-x)(6+x)}$$

$$(e) \quad y^2 - 5y + 12 = \boxed{\text{not factorable}}$$

product = c = 12 < $\begin{matrix} - \\ - \end{matrix}$
 sum = b = -5
 $12 = 4 \cdot 3 = 6 \cdot 2 = 12 \cdot 1$

$$(f) \quad 2x^6 + 8x^5 - 42x^4 =$$

$$2x^4(x^2 + 4x - 21) =$$

$$\boxed{2x^4(x+7)(x-3)}$$

product = c = -21 < $\begin{matrix} +7 \\ -3 \end{matrix}$
 sum = b = 4
 $21 = 7 \cdot 3$

$$(g) \quad d^2 + 4d - 45 = \boxed{(d+9)(d-5)}$$

product = c = -45 < $\begin{matrix} +9 \\ -5 \end{matrix}$
 sum = b = 4
 $45 = 9 \cdot 5$

$$(h) \quad 3y^2 - 48y + 192 =$$

$$3(y^2 - 16y + 64) =$$

$$3(y^2 - 16y + 8^2) =$$

$$\boxed{3(y-8)^2}$$

$$(i) \quad m^2 - 13m - 30 = \boxed{(m+2)(m-15)}$$

product = c = -30 < $\begin{matrix} +2 \\ -15 \end{matrix}$
 sum = b = -13
 $30 = 3 \cdot 10 = 6 \cdot 5 = 15 \cdot 2$

$$(j) \quad x^3 - 64 = x^3 - 4^3 \text{ difference of cubes}$$

$$= (x-4)(x^2 + 4x + 4^2)$$

$$= \boxed{(x-4)(x^2 + 4x + 16)}$$

$$(k) \quad 3t^2 - 11t - 20$$

split the middle term
 product = ac = 3(-20) = -60 < $\begin{matrix} +4 \\ -15 \end{matrix}$
 sum = b = -11
 $60 = 4 \cdot 15$

$$3t^2 - 11t - 20 = 3t^2 + 4t - 15t - 20$$

$$= t(3t+4) - 5(3t+4)$$

$$= \boxed{(3t+4)(t-5)}$$

$$(l) \quad a^2 + 16 = \boxed{\text{not factorable}}$$

$$(m) \quad 2x^2 - 10x + 3xy - 15y =$$

$$2x(x-5) + 3y(x-5) =$$

$$\boxed{(x-5)(2x+3y)}$$

(4) (a) $9x^3 - 49x = 0$
 $x(9x^2 - 49) = 0$
 $x((3x)^2 - 7^2) = 0$

$x(3x-7)(3x+7) = 0$
 zero-factor property \Rightarrow
 $x=0$ OR $3x-7=0$ OR $3x+7=0$
 $x = \frac{7}{3}$ $x = -\frac{7}{3}$

b. $x \in \{0, \frac{7}{3}, -\frac{7}{3}\}$

(b) $3x(x+1) = 2x+2$
 $3x^2 + 3x - 2x - 2 = 0$
 $3x^2 + x - 2 = 0$

split the middle term $+3$
 product = $ac = 3(-2) = -6 < -2$
 sum = $b = 1$
 $6 = 3 \cdot 2$

$3x^2 + 3x - 2x - 2 = 0$
 $3x(x+1) - 2(x+1) = 0$
 $(x+1)(3x-2) = 0$

zero-factor property \Rightarrow
 $x+1=0$ OR $3x-2=0$
 $x = -1$ $x = \frac{2}{3}$

b. $x \in \{-1, \frac{2}{3}\}$

(c) $a^2 + (a+3)^2 = 225$
 $a^2 + a^2 + 6a + 9 - 225 = 0$
 $2a^2 + 6a - 216 = 0 \quad /: 2$
 $a^2 + 3a - 108 = 0$

product = $c = -108 < -9$
 sum = $b = 3$
 $108 = 9 \cdot 12$

$(a+12)(a-9) = 0$
 zero-factor property \Rightarrow
 $a+12=0$ OR $a-9=0$
 $a = -12$ $a = 9$

$a \in \{-12, 9\}$

(d) $(y-4)(y-2) = 15$
 $y^2 - 2y - 4y + 8 - 15 = 0$
 $y^2 - 6y - 7 = 0$

$(y-7)(y+1) = 0$
 $y-7=0$ OR $y+1=0$
 $y = 7$ $y = -1$

$y \in \{7, -1\}$

(5) $d = 16t^2 + 40t$
 $t = \text{time (seconds)}$
 $d = \text{distance (ft)}$
 find t if $d = 144 \text{ ft}$

Sub: $144 = 16t^2 + 40t$
 (quadratic equation)
 We'll solve it by factoring:
 $16t^2 + 40t - 144 = 0 \quad /: 8$
 $2t^2 + 5t - 18 = 0$

split middle term $+9$
 product = $ac = -36 < -9$
 sum = $b = 5$
 $36 = 9 \cdot 4$

$2t^2 + 9t - 4t - 18 = 0$
 $t(2t+9) - 2(2t+9) = 0$
 $(2t+9)(t-2) = 0 \Rightarrow$
 $2t+9=0$ OR $t-2=0$

$t = -\frac{9}{2}$
 not possible $t = 2$ seconds
 The stone will travel 144 ft in 2 seconds

QUIZ #2 @ 85 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Simplify the following expressions (if possible). Write the final answer using only positive exponents (if necessary).

a) $a^{-2} + 2a^{-2}$	d) $\frac{(2x^{-1}y)^2}{x^{-3}y^5}$	g) $\left(9y + \frac{2}{3}\right)\left(9y - \frac{2}{3}\right)$
b) $x^2 - 2x^3$	e) $3^{-2} + \left(\frac{1}{2}\right)^{-1} + (-2)^{-1} + 5^0$	h) $2x(5x-1)(5x+1)$
c) $t^{-1} - t^{-1}$	f) $(y^3 - 5)^2$	i) $(t^{-1})(-t^{-1})$
		j) $\left(\frac{1}{3}a - c\right)^2$

2. Do the following division: $\frac{x^4 - 4x^3 + 5x^2 - 3x + 2}{x^2 + 3}$

3. Factor each expression as completely as possible. If prime, state so.

a) $2x - 6 - ax + 3a$	e) $y^2 - 5y + 12$	i) $m^2 - 13m - 30$
b) $3(5-x) + y(5-x)$	f) $2x^6 + 8x^5 - 42x^4$	j) $x^3 - 64$
c) $m^2 + 9m + 14$	g) $d^2 + 4d - 45$	k) $3t^2 - 11t - 20$
d) $36 - x^2$	h) $3y^2 - 48y + 192$	l) $a^2 + 16$
		m) $2x^2 - 10x + 3xy - 15y$

4. Solve each equation by the factoring method:

a) $9x^3 - 49x = 0$	c) $a^2 + (a+3)^2 = 225$
b) $3x(x+1) = 2x+2$	d) $(y-4)(y-2) = 15$

5. A stone is thrown downward off a cliff so that the distance it has traveled after t seconds is given by $d = 16t^2 + 40t$, where d is measured in feet. How long will it take the stone to travel 144 feet?