Math 51 Summer 2010 Name:

QUIZ #3 @ 85 points

Write in a neat and organized fashion. <u>Write your complete solutions on SEPARATE PAPER.</u> For an exercise to be complete there needs to be a detailed solution to the problem. No proof, no credit given! Clearly label each exercise.

1. Do the following operations and simplify. For (k) and (l), write answers in exponential form with only positive exponents.

a) $4\sqrt{50}+3\sqrt{12}-5\sqrt{45}$

e) $\left(5\sqrt{7}-2\sqrt{3}\right)^2$

i) $\left(\sqrt{x}+\sqrt{3x-1}\right)^2$

b) $\left(5\sqrt{7}-2\sqrt{3}\right)\left(3\sqrt{7}+4\sqrt{3}\right)$

f) $5\sqrt{5}\left(3\sqrt{5}+\sqrt{2}\right)-\sqrt{2}\left(2\sqrt{8}+\sqrt{5}\right)$

j) $\dfrac{12-4\sqrt{10}}{4}$

c) $2\sqrt{12}+3\sqrt{75}$

g) $\left(\sqrt{6}+1\right)^2$

k) $\dfrac{3^{-\frac{3}{4}}\cdot 3^{\frac{5}{4}}}{3^{-\frac{1}{4}}}$

d) $2\sqrt[3]{32m^3}-\sqrt[3]{108m^3}$

h) $\dfrac{16+8\sqrt{2}}{24}$

l) $\dfrac{\left(a^2b^{\frac{1}{2}}\right)^{-1}}{a^{-\frac{1}{2}}b^3}$

2. Solve the following equations.

a) $5\sqrt{x}=\sqrt{10x+15}$

c) $\sqrt{2a+11}+\sqrt{a+6}=2$

b) $y=\sqrt{y^2-2y-6}$

d) $\sqrt{5x+11}-x=3$

3) Rationalize each denominator and simplify.

a) $\dfrac{2}{\sqrt{3}}$

b) $\dfrac{5}{\sqrt[3]{2}}$

c) $\dfrac{1}{4+\sqrt{15}}$

$\boxed{\text{Quiz 3 – Solutions}}$

(1)

(a) $4\sqrt{50}+3\sqrt{12}-5\sqrt{45}=$

$4\sqrt{25\cdot2}+3\sqrt{4\cdot3}-5\sqrt{9\cdot5}=$

$4\cdot5\sqrt{2}+3\cdot2\sqrt{3}-5\cdot3\sqrt{5}=$

$\boxed{20\sqrt{2}+6\sqrt{3}-15\sqrt{5}}$

(b) $(5\sqrt{7}-2\sqrt{3})(3\sqrt{7}+4\sqrt{3})=$

$15(\sqrt{7})^2+20\sqrt{21}-6\sqrt{21}-8(\sqrt{3})^2=$

$15\cdot7+14\sqrt{21}-8(3)=$

$105+14\sqrt{21}-24=\boxed{81+14\sqrt{21}}$

(c) $2\sqrt{12}+3\sqrt{75}=$

$2\sqrt{4\cdot3}+3\sqrt{25\cdot3}=$

$2\cdot2\sqrt{3}+3\cdot5\sqrt{3}=4\sqrt{3}+15\sqrt{3}=\boxed{19\sqrt{3}}$

(d) $2\sqrt[3]{32m^3}-\sqrt[3]{108m^3}=$

$2\sqrt[3]{8\cdot4m^3}-\sqrt[3]{27\cdot4m^3}=$

$2\cdot2\sqrt[3]{4m^3}-3\sqrt[3]{4m^3}=\boxed{\sqrt[3]{4m^3}}$

(e) $(5\sqrt{7}-2\sqrt{3})^2=$

$(5\sqrt{7})^2-2(5\sqrt{7})(2\sqrt{3})+(2\sqrt{3})^2=$

$25\cdot7-20\sqrt{21}+4\cdot3=$

$175-20\sqrt{21}+12=\boxed{187-20\sqrt{21}}$

(f) $5\sqrt{5}(3\sqrt{5}+\sqrt{2})-\sqrt{2}(2\sqrt{8}+\sqrt{5})=$

$15(\sqrt{5})^2+5\sqrt{10}-2\sqrt{16}-\sqrt{10}=$

$15\cdot5+5\sqrt{10}-2\cdot4-\sqrt{10}=$

$75+4\sqrt{10}-8=\boxed{67+4\sqrt{10}}$

(g) $(\sqrt{6}+1)^2=(\sqrt{6})^2+2\sqrt{6}+1^2$

$=6+2\sqrt{6}+1=\boxed{7+2\sqrt{6}}$

(h) $\dfrac{16+8\sqrt{2}}{24}=\dfrac{8(2+\sqrt{2})}{24}=\boxed{\dfrac{2+\sqrt{2}}{3}}$

(i) $(\sqrt{x}+\sqrt{3x-1})^2=$

$(\sqrt{x})^2+2\sqrt{x}\cdot\sqrt{3x-1}+(\sqrt{3x-1})^2=$

$x+2\sqrt{x(3x-1)}+3x-1=$

$\boxed{4x+2\sqrt{x(3x-1)}-1}$

(j) $\dfrac{12-4\sqrt{10}}{4}=\dfrac{4(3-\sqrt{10})}{4}=\boxed{3-\sqrt{10}}$

(k) $\dfrac{3^{-\frac{3}{4}}\cdot3^{\frac{5}{4}}}{3^{-\frac{1}{4}}}=\dfrac{3^{-\frac{3}{4}+\frac{5}{4}}}{3^{-\frac{1}{4}}}=$

$=\dfrac{3^{\frac{2}{4}}}{3^{-\frac{1}{4}}}=3^{\frac{2}{4}-(-\frac{1}{4})}=3^{\frac{2}{4}+\frac{1}{4}}=\boxed{3^{\frac{3}{4}}}$

(l) $\dfrac{(a^2b^{\frac{1}{2}})^{-1}}{a^{-\frac{1}{2}}b^3}=\dfrac{(a^2)^{-1}(b^{\frac{1}{2}})^{-1}}{a^{-\frac{1}{2}}b^3}$

$=\dfrac{a^{-2}b^{-\frac{1}{2}}}{a^{-\frac{1}{2}}b^3}=a^{-2-(-\frac{1}{2})}b^{-\frac{1}{2}-3}$

$=a^{-2+\frac{1}{2}}b^{-\frac{7}{2}}=a^{-\frac{3}{2}}b^{-\frac{7}{2}}$

$=\boxed{\dfrac{1}{a^{3/2}b^{7/2}}}$

(2) (a) $5\sqrt{x} = \sqrt{10x+15}$ $/^2$ (radical eq)

$(5\sqrt{x})^2 = (\sqrt{10x+15})^2$

$25x = 10x+15$ (linear eq.)

$25x - 10x = 15$

$15x = 15 \Rightarrow x = 1$

Check $5\sqrt{1} \stackrel{?}{=} \sqrt{10\cdot 1+15}$

$5 = \sqrt{25}$ TRUE

Therefore, $\boxed{x \in \{1\}}$

(b) $y = \sqrt{y^2-2y-6}$ $/^2$ (radical eq)

$y^2 = (\sqrt{y^2-2y-6})^2$

$\cancel{y^2} = \cancel{y^2} - 2y - 6$

$0 = -2y - 6$ (linear eq.)

$2y = -6 \Rightarrow y = -3$

check $-3 = \sqrt{(-3)^2 - 2(-3) - 6}$

not possible, as $\sqrt{n} \geq 0$ for any $n \geq 0$

Therefore, $\boxed{y \in \phi}$ (No solutions)

(c) $\sqrt{2a+11} + \sqrt{a+6} = 2$ (radical eq)

$\sqrt{2a+11} = 2 - \sqrt{a+6}$ $/^2$

$(\sqrt{2a+11})^2 = (2-\sqrt{a+6})^2$

$2a+11 = 4 - 2\cdot 2\sqrt{a+6} + (\sqrt{a+6})^2$

$2a+11 = 4 - 4\sqrt{a+6} + a + 6$

$2a+11 = 10 - 4\sqrt{a+6} + a$ (radical equation)

$2a + 11 - 10 - a = -4\sqrt{a+6}$

$a+1 = -4\sqrt{a+6}$ $/^2$

$(a+1)^2 = (-4\sqrt{a+6})^2$

$a^2+2a+1 = 16(a+6)$

$a^2+2a+1 = 16a + 96$ (quadratic eq.)

solve by factoring:

$a^2+2a+1-16a-96 = 0$

$a^2 - 14a - 95 = 0$

product $= c = -95 \left< \begin{matrix} +5 \\ -19 \end{matrix} \right.$

sum $= b = -14$

$95 = 5\cdot 19$

$(a+5)(a-19) = 0 \Rightarrow$

$a+5 = 0$ OR $a - 19 = 0$

$a = -5$ $\quad a = 19$

check

$a=-5$: $\sqrt{2(-5)+11} + \sqrt{-5+6} \stackrel{?}{=} 2$

$\sqrt{1} + \sqrt{1} = 2$

$1+1 = 2$ TRUE

$\Rightarrow a = -5$ is a solution

$a = 19$: $\sqrt{2(19)+11} + \sqrt{19+6} \stackrel{?}{=} 2$ FALSE

$\Rightarrow a = 19$ not a solution

Therefore, $\boxed{a \in \{-5\}}$

(d) $\sqrt{5x+11} - x = 3$

radical equation

$\sqrt{5x+11} = 3+x \mid)^2$

$(\sqrt{5x+11})^2 = (3+x)^2$

$5x+11 = 9+6x+x^2$

quadratic eq.

Solve by factoring:

$x^2+6x+9-5x-11=0$

$x^2+x-2=0$

$(x+2)(x-1)=0 \Rightarrow$

$x+2=0$ OR $x-1=0$

$x=-2$ $x=1$

<u>check</u>

$x=-2$: $\sqrt{5(-2)+11} - (-2) \stackrel{?}{=} 3$

$\sqrt{1}+2=3$

$1+2=3$ TRUE

so $x=-2$ is a solution

$x=1$: $\sqrt{5\cdot 1+11} - 1 \stackrel{?}{=} 3$

$\sqrt{16} - 1 = 3$

$4-1=3$ TRUE

so $x=1$ is a solution

Therefore, $\boxed{x \in \{-2, 1\}}$

(3) (a) $\dfrac{\sqrt{3}}{2} \Big/ \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{(\sqrt{3})^2} = \boxed{\dfrac{2\sqrt{3}}{3}}$

(b) $\dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} \cdot \dfrac{5}{\sqrt[3]{2}} = \dfrac{5\sqrt[3]{4}}{\sqrt[3]{2}\,\sqrt[3]{4}} = \dfrac{5\sqrt[3]{4}}{\sqrt[3]{8}}$

$= \boxed{\dfrac{5\sqrt[3]{4}}{2}}$

(c) $\dfrac{4-\sqrt{15}}{4-\sqrt{15}} \cdot \dfrac{1}{4+\sqrt{15}} = \dfrac{4-\sqrt{15}}{(4+\sqrt{15})(4-\sqrt{15})}$

$= \dfrac{4-\sqrt{15}}{4^2-(\sqrt{15})^2} = \dfrac{4-\sqrt{15}}{16-15}$

$= \boxed{4-\sqrt{15}}$