

REVIEW TEST 2

Chapters 3 (3.1 – 3.10)& 4 (4.1 – 4.6)

To prepare for the test, you should:

- study **all quizzes** and all **examples done in class**, as well as your **homework** from the listed sections.
- know how to prove formally the following theorems or properties:
 - Section 3.4
 - Rule page 182 (Derivative of sine)
 - Rule page 183 (Derivative of cosine)
 - Section 3.7
 - Rule page 216 (Derivative of a^x)
 - Derivative of the Natural Logarithmic Function using implicit differentiation or using The Derivative Rule for Inverses (see page 214 – 215)
 - Theorem 6 (The Number e as a limit)
 - Section 4.2
 - Corollary 1 (Functions with zero derivatives are constant)
 - Corollary 2 (Functions with the same derivative differ by a constant)
- Handout 3.3 & 3.9 – all examples and exercises
- Handout 4.1 & 4.2 – all exercises
- Handout 4.5 – all exercises
- know the following:
 - the derivative of a function using the definition
 - the differentiation rules for polynomials, exponential, products, and quotients
 - various instantaneous rates of change (velocity, acceleration, marginal cost, etc)
 - the differentiation rule for trigonometric functions and their inverses, as well as for exponential and logarithmic functions
 - The Chain Rule
 - how to use implicit differentiation to find the derivative of a function
 - derivatives of higher order
 - logarithmic differentiation
 - related rates application
 - definition of a critical point
 - The Closed Interval Method
 - The Second Derivative Test
 - Definition of an inflection point
 - The Increasing and Decreasing Test
 - How to find the linearization of a function at a point

Note: Please check website for Handouts and their solutions.

- **More practice Chapter 3 – Practice Exercises page 255 (1 – 83 odd, 99, 101, 103, 127 – 133 odd, 137 – 141 odd)**
- **More practice Chapter 4:**

4.1 Finding Critical Numbers. Finding Absolute Minimum and Maximum Values of a Function

4.4 Graphing a Function

Definition A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

The Closed Interval Method

To find the absolute minimum and maximum values of a continuous function f on a closed interval $[a,b]$:

1. Find the critical numbers of f .
2. Find the values of f at the critical numbers and at the endpoints of the interval.
3. The largest of the values is the absolute maximum value; the smallest of the values is the absolute minimum value.

Exercise 1 Find the critical numbers of each function:

- | | | |
|----------------------------------|------------------------------------|------------------------|
| a) $f(x) = x^{\frac{3}{5}}(4-x)$ | d) $f(x) = x^{\frac{4}{5}}(x-4)^2$ | |
| b) $f(r) = \frac{r}{r^2+1}$ | e) $F(x) = \sqrt[3]{x^2-x}$ | g) $g(q) = q + \sin q$ |
| c) $f(z) = \frac{z+1}{z^2+z+1}$ | f) $f(q) = \sin^2(2q)$ | h) $f(x) = x \ln x$ |

Exercise 2 Find the absolute minimum and maximum values of each function on the given interval:

- | | |
|--|--|
| a) $f(x) = x - 2\sin x, x \in [0, 2p]$ | d) $f(x) = \sin x + \cos x, x \in \left[0, \frac{p}{3}\right]$ |
| b) $f(x) = \sqrt{9-x^2}, x \in [-1, 2]$ | e) $f(x) = x - 2\cos x, x \in [-p, p]$ |
| c) $f(x) = x^2 + \frac{2}{x}, x \in \left[\frac{1}{2}, 2\right]$ | f) $f(x) = x - 2\sin x, x \in [0, 3p]$ |

Exercise 3 Graph each function (as we did in class):

- | | |
|------------------------------------|---|
| a) $f(x) = 2 - 2x - x^3$ | d) $f(x) = 2\cos x + \sin^2 x, x \in [-p, p]$ |
| b) $f(x) = \frac{x}{(1+x)^2}$ | e) $f(x) = \frac{1+x^2}{1-x^2}$ |
| c) $f(x) = \frac{\ln x}{\sqrt{x}}$ | |

4.6 L'Hopital's Rule Suppose f and g are differentiable and $g'(x) \neq 0$ near a (except possibly at a). Suppose that

$$\lim_{x \rightarrow a} f(x) = 0 \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = 0$$

or that

$$\lim_{x \rightarrow a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$). Then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

Exercise 4 Find each limit. Use l'Hopital's Rule where appropriate. (i)
If there is a more elementary method, consider it. (ii)
If l'Hopital's Rule does not apply, explain why. (iii)

a) $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$

g) $\lim_{x \rightarrow 0^+} x^{\sin x}$

m) $\lim_{x \rightarrow \infty} \left(x e^{\frac{1}{x}} - x \right)$

b) $\lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1}$

h) $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$

n) $\lim_{x \rightarrow 0} (1 - 2x)^{\frac{1}{x}}$

c) $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$

i) $\lim_{x \rightarrow \infty} e^{-x} \ln x$

o) $\lim_{x \rightarrow 1^+} (x - 1) \tan\left(\frac{px}{2}\right)$

d) $\lim_{x \rightarrow 0} \frac{\tan px}{\tan qx}$

j) $\lim_{x \rightarrow \infty} x^{\frac{\ln 2}{1 + \ln x}}$

p) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x - 1} \right)$

e) $\lim_{x \rightarrow -\infty} x^2 e^x$

k) $\lim_{x \rightarrow 0} \frac{1 - e^{-2x}}{\sec x}$

r) $\lim_{x \rightarrow 0^+} (-\ln x)^x$

f) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$

l) $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$

s) $\lim_{x \rightarrow 0^+} x^2 \ln x$

Answers

Exercise 1: a) 0, 3/2; b) ± 1 ; c) 0, -2; d) 0, 8/7, 4; e) 0, 1/2, 1; f) $k\mathbf{p}/4$, k integer; g) $(2k + 1)\mathbf{p}$, k integer; h) 1/e; i)

Exercise 2: a) abs. min: $f\left(\frac{\mathbf{p}}{3}\right) = \frac{\mathbf{p}}{3} - \sqrt{3}$, abs. max: $f\left(\frac{5\mathbf{p}}{3}\right) = \frac{5\mathbf{p}}{3} + \sqrt{3}$; b) abs. max: $f(0) = 3$, abs.

min: $f(2) = \sqrt{5}$; c) abs. max: $f(2) = 5$, abs. min: $f(1) = 3$; d) abs. max: $f\left(\frac{\mathbf{p}}{4}\right) = \sqrt{2}$, abs. min: $f(0) = 1$; e)

abs. max: $f(\mathbf{p}) = \mathbf{p} + 2$, abs. min: $f\left(-\frac{\mathbf{p}}{6}\right) = -\frac{\mathbf{p}}{6} - \sqrt{3}$; f) abs. min: $f\left(\frac{\mathbf{p}}{3}\right) = \frac{\mathbf{p}}{3} - \sqrt{3} \approx -0.68$, abs. max:

$$f(3\mathbf{p}) = 3\mathbf{p}$$

Exercise 4 a) ii -2; b) i a/b; c) i 0; d) i p/q; e) i 0; f) i 1; g) 1; h) i $\frac{n^2 - m^2}{2}$; i) i 0; j) 2; k) iii 0; l) i 0; m) 1; n) e^{-2} ;

o) i $-2/\mathbf{p}$; p) i 1/2; r) 1; s) i 0.