REVIEW TEST 1
Chapter 2 & Sections 3.1, 3.2

Chapter 2 – Limits and Continuity

Section 3.1 – The Derivative as a Function

Section 3.2 – Differentiation Rules for Polynomials, Exponentials, Products, and Quotients

After completing these sections, you should:

- be able to find the rate of change of a function over an interval
- know the definition of the limit of a function at a point
- be able to find limits given a graph
- be able to calculate limits using The Limit Laws
- know and be able to apply The Sandwich (Squeeze) Theorem in finding limits
- be able to calculate one-sided limits and infinite limits
- know the definition of a function continuous at a point
- be able to recognize points of discontinuity
- be able to find the slope of the tangent to a curve at a given point
- be able to find an equation of the tangent line to a graph at a point
- be able to find vertical and horizontal tangents to a graph
- be able to recognize the special cases $\frac{0}{0}, 0\cdot\infty, \infty-\infty, \frac{\infty}{\infty}$
- know how to find special – case limits
- be able to find the derivative of a function using the definition
- know the differentiation rules for polynomials, exponential, products, and quotients
- be able to find higher-order derivatives

Know how to prove formally the following theorems or properties:

- Section 2.4
    - Theorem 7 ($\lim_{q\to 0}\frac{\sin q}{q}=1$)
- Section 3.1
    - Theorem 1 (Differentiability implies continuity)
- Section 3.2
    - Rule 1 (Derivative of a constant)
    - Rule 3 (Constant multiple rule)
    - Rule 4 (Derivative sum rule)
    - Rule 5 (Derivative product rule)

Know how to prove the following special limits:

- $\lim_{x\to\infty}\frac{\sin x}{x}=0$ and $\lim_{x\to\infty}\frac{\cos x}{x}=0$
- $\lim_{x\to 0}x^n\sin\frac{1}{x}=0$ and $\lim_{x\to 0}x^n\sin\frac{1}{x}=0$ for $\forall n\in\mathbb{N}$.
- $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ (Example 5, section 2.4)

To prepare for the test, you should study **all quizzes** and all **examples done in class**, as well as your **homework** (check website for solutions to selected homework exercises).

More practice:

Textbook – Practice Exercises page 139 : # 1, 3, 9 – 23 odd, 29 – 39 odd

1. Let $f(x)=\begin{cases}\sqrt{-x}, x<0\\ 3-x, 0\le x<3.\\ (x-3)^2, x\ge 3\end{cases}$

a) Evaluate each limit, if it exists. (i) $\lim_{x\to 0^+} f(x)$ (ii) $\lim_{x\to 0^-} f(x)$ (iii) $\lim_{x\to 0} f(x)$ (iv) $\lim_{x\to 3^-} f(x)$ (v) $\lim_{x\to 3^+} f(x)$ (vi) $\lim_{x\to 3} f(x)$

b) Where is *f* discontinuous?

2. Find an equation of the tangent line to the curve $y=x^3-2x$ at the point $(2,4)$ in two ways: a) using the definition; b) using the differentiation rules.

3. Let $f(x)=\begin{cases}x+1, x\le a\\ x^2, x>a\end{cases}$ Find an a such that *f* is continuous everywhere.

4-11. Find the limits.

4. $\lim_{x\to 1} e^{x^3-x}$ 5. $\lim_{x\to -3}\dfrac{x^2-9}{x^2+2x-3}$ 6. $\lim_{h\to 0}\dfrac{(h-1)^3+1}{h}$ 7. $\lim_{r\to 9}\dfrac{\sqrt{r}}{(r-9)^4}$

8. $\lim_{v\to 4^+}\dfrac{4-v}{|4-v|}$ 9. $\lim_{x\to\infty} e^{-3x}$ 10. $\lim_{x\to 0}\dfrac{1-\sqrt{1-x^2}}{x}$ 11. $\lim_{x\to\infty}\dfrac{\sqrt{3x^2-1}}{x-1}$

12. Find $\lim_{x\to 0}\sqrt{x^3+x^2}\sin\dfrac{p}{x}$. **13.** Prove that $\lim_{x\to 0} x^4\cos\dfrac{2}{x}=0$ **14.** Prove that $\lim_{x\to 0^+}\sqrt{x}e^{\sin(p/x)}=0$

15-16. Find the limit, if it exists. If the limit does not exist, explain way.

15. $\lim_{x\to -4}|x+4|$ 16. $\lim_{x\to 2}\dfrac{|x-2|}{x-2}$

17 – 27 Find the limits.

17 $\lim_{x\to 5^-}\dfrac{e^x}{(x-5)^3}$ 18. $\lim_{x\to\infty}\dfrac{x+2}{\sqrt{9x^2+1}}$ 19. $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$ 20. $\lim_{x\to\infty}\dfrac{\sin^2 x}{x^2}$

21. $\lim_{x\to\infty}\cos x$ 22. $\lim_{x\to -\infty} x^3-5x^2$ 23. $\lim_{x\to 0}\dfrac{\sin^2 x}{x}$

24. $\lim_{x\to -1}\dfrac{\sin(x^2-x-2)}{x+1}$ 25. $\lim_{x\to 1}\dfrac{\sin(1-\sqrt{x})}{x-1}$ 26. $\lim_{t\to 0}\dfrac{\sin(1-\cos t)}{1-\cos t}$ 27. $\lim_{x\to\pm\infty} x\sin\dfrac{1}{x}$

28. Find $\lim_{x\to\infty} f(x)$ if $\frac{4x-1}{x} < f(x) < \frac{4x^2+3x}{x^2}$ for all $x>5$.

29 - 34 Differentiate the function.

29. $y = ae^v + \frac{b}{v} + \frac{c}{v^2}$, find $\frac{dy}{dv}$

30. $y = \frac{x^2+4x+3}{\sqrt{x}}$, find $\frac{dy}{dx}$

31. $y = \frac{t^2}{3t^2-2t+1}$, find $\frac{dy}{dt}$

32. $f(x) = \frac{x}{x+\frac{c}{x}}$

33. $y = \frac{1}{s+ke^s}$, find $\frac{dy}{ds}$

34. $P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$, find $\frac{dP}{dV}$

35. Find the first and second derivatives of the function $G(r) = \sqrt{r} + \sqrt[3]{r}$.

36. Find equations of both lines through the point $(2,-3)$ that are tangent to the parabola $y = x^2 + x$.

37. Find a second-degree polynomial P such that $P(2)=5$, $P'(2)=3$, and $P''(2)=2$.

38. Find $\lim_{x\to 1} \frac{x^{1000}-1}{x-1}$.

39. If f is a differentiable function, find an expression for the derivative of each of the following functions:

a) $y = x^2 f(x)$ b) $y = \frac{f(x)}{x^2}$ c) $y = \frac{1+xf(x)}{\sqrt{x}}$

40. Find equations of the tangent lines to the curve $y = \frac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$.

Answers

1) a) (i) 3 (ii) 0 (iii) DNE (iv) 0 (v) 0 (vi) 0 ; b) 0, -3

2) $y-4=10(x-2)$; 3) $\frac{1\pm\sqrt{5}}{2}$; 4) 1; 5) $\frac{3}{2}$; 6) 3; 7) ∞; 8) -1; 9) 0; 10) 0; 11) $\sqrt{3}$; 12) 0; 13); 14); 15) 0

16) DNE; 17) $-\infty$; 18) $\frac{1}{3}$; 19) $\frac{1}{6}$; 20) 0; 21) DNE; 22) $-\infty$; 23) 0; 24) -3; 25) $-\frac{1}{2}$; 26) 1; 27) 1; 28) 4;

29) $y' = ae^v - \frac{b}{v^2} - \frac{2c}{v^3}$; 30) $y' = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x\sqrt{x}}$; 31) $y' = \frac{2t(1-t)}{(3t^2-2t+1)^2}$; 32) $f'(x) = \frac{2cx}{(x^2+c)^2}$;

33) $-\frac{1+ke^s}{(s+ke^s)^2}$; 34) $3\frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$; 35) $G'(r) = \frac{1}{2\sqrt{r}} + \frac{1}{3\sqrt[3]{r^2}}$, $G''(r) = -\frac{1}{4\sqrt{r^3}} - \frac{2}{9\sqrt[3]{r^5}}$; 36) $y=-x-1$

and $y = 11x-25$; 37) $P(x) = x^2 - x + 3$; 38) 1000; 39) a) $y' = x^2 f'(x) + 2xf(x)$, b) $y' = \frac{xf'(x) - 2f(x)}{x^3}$,

c) $y' = \frac{xf(x) + 2x^2 f'(x) - 1}{2x^{3/2}}$; 40) $y = \frac{1}{2}x - \frac{1}{2}$ and $y = \frac{1}{2}x + \frac{7}{2}$.