MATH 51

TEST 1 - SOLUTIONS

① (a) $-\frac{4}{5} + 3m + \frac{1}{5}m - 5 - \frac{8}{5}m =$

$= \frac{-4}{5} - \frac{5}{1} + \frac{3m}{1} + \frac{m}{5} - \frac{8m}{5}$

$= \frac{-4-25}{5} + \frac{15m + m - 8m}{5}$

$= \boxed{\frac{-29}{5} + \frac{8}{5}m}$

(b) $5(a-1) - 4[2a - 4(a-3)] =$

$= 5a - 5 - 4(2a - 4a + 12)$

$= 5a - 5 - 4(-2a + 12)$

$= 5a - 5 + 8a - 48$

$= \boxed{13a - 53}$

② $-xy + x^2$ for $x = -\frac{2}{3}, y = \frac{9}{4}$

$-xy + x^2 = -\left(\frac{-2}{3}\right)\left(\frac{9}{4}\right) + \left(\frac{-2}{3}\right)^2$

$= \frac{3}{2} + \frac{4}{9}$ $\quad LCD = 18$

$= \frac{27}{18} + \frac{8}{18}$

$= \boxed{\frac{35}{18}}$

③ (a) $-2(3y-1) - 2y = 4(3-y) + 2y$

$-6y + 2 - 2y = 12 - 4y + 2y$

$-8y + 2 = 12 - 2y$

$2 - 12 = -2y + 8y$

$-10 = 6y$

$y = \frac{-10}{6} = \frac{-5}{3}$

$\boxed{y = \frac{-5}{3}}$

(b) $\frac{2}{5}x + \frac{1}{10}x - 18 = \frac{1}{20}x$

Method I - isolate the variable

$\frac{2}{5}x + \frac{1}{10}x - \frac{1}{20}x = 18$

$LCD = 20$

$\frac{8}{20}x + \frac{2}{20}x - \frac{1}{20}x = 18$

$\frac{9}{20}x = 18 \quad \Big/ \cdot \frac{20}{9}$

$x = \frac{18}{1} \cdot \frac{20}{9}$

$\boxed{x = 40}$

Method II - eliminate the fractions

$\frac{2}{5}x + \frac{1}{10}x - \frac{18}{1} = \frac{1}{20}x$

$LCD = 20$

$\frac{8}{20}x + \frac{2}{20}x - \frac{360}{20} = \frac{1}{20}x \quad \Big/ \cdot 20$

$8x + 2x - 360 = x$

$10x - 360 = x$

$10x - x = 360$

$9x = 360 \Rightarrow x = \frac{360}{9} = 40$

$x = 40$

(c) $8(a-3) + 4a = 6(2a+1) - 10$

$8a - 24 + 4a = 12a + 6 - 10$

$\cancel{12a} - 24 = \cancel{12a} - 4$

$24 = -4$ contradiction

$\Rightarrow$ No solutions

$\boxed{a \in \emptyset}$

(d) $\dfrac{3x-2}{5} = \dfrac{4-x}{3}$

Cross-product property:

$3(3x-2) = 5(4-x)$

$9x - 6 = 20 - 5x$

$9x + 5x = 20 + 6$

$14x = 26$

$x = \dfrac{26}{14} = \dfrac{13}{7}$

$\boxed{x = \dfrac{13}{7}}$

(e) $9(6+x) = -7(2+x)$

$54 + 9x = -14 - 7x$

$9x + 7x = -14 - 54$

$16x = -68$

$x = \dfrac{-68}{16} = \dfrac{-17}{4}$

$\boxed{x = \dfrac{-17}{4}}$

(f) $\dfrac{12c}{5} = \dfrac{c}{3}$

Cross-product property:

$3(12c) = 5(c)$

$36c = 5c$

$36c - 5c = 0$

$31c = 0$

$\boxed{c = 0}$

(g) $ax + by = c \quad , y = ?$

$by = -ax + c \quad \Big/ \div b$

$\boxed{y = \dfrac{-a}{b}x + \dfrac{c}{b}}$

(h) $C = \dfrac{5}{9}(F - 32) \quad \Big/ \cdot \dfrac{9}{5} \quad , F = ?$

$\dfrac{9}{5}C = F - 32$

$\dfrac{9}{5}C + 32 = F$

$\boxed{F = \dfrac{9}{5}C + 32}$

(i) $A = \dfrac{1}{2}h(b + B) \quad \Big/ \cdot \dfrac{2}{h} \quad , B = ?$

$\dfrac{2A}{h} = b + B$

$\dfrac{2A}{h} - b = B$

$\boxed{B = \dfrac{2A}{h} - b}$

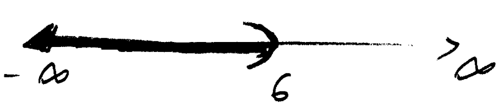
(4) (a) $3 + \dfrac{2}{3}p < 7$

$\dfrac{2}{3}p < 7 - 3$

$\dfrac{2}{3}p < 4 \quad \Big/ \cdot \dfrac{3}{2}$

$p < \dfrac{\cancel{4}^{2}}{1} \cdot \dfrac{3}{\cancel{2}}$

(i) $\boxed{p < 6}$

(ii)


(iii) $p \in (-\infty, 6)$

(b) $-3 < -x + 7 \le 15$

$\quad -7 \quad\quad -7 \quad\quad -7$

$-10 < -x \le 8 \quad /\cdot(-1)$

$10 > x \ge 8$

(i) $\boxed{8 \le x < 10}$

(ii) $-\infty \quad 8 \quad 10 \quad \infty$

(iii) $x \in [8, 10)$

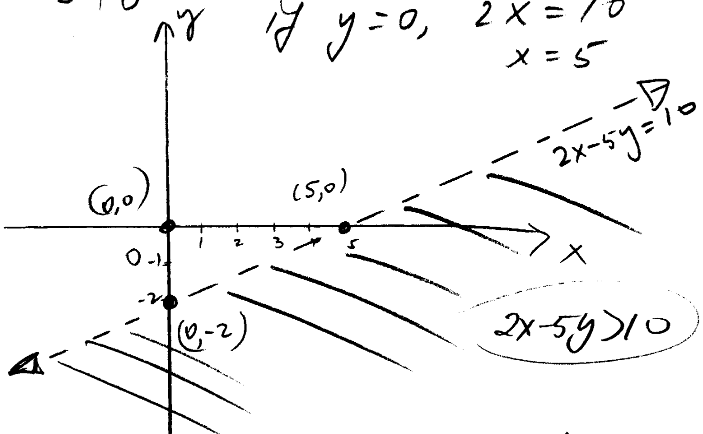
(5) $2x - 5y > 10$

Boundary line: $2x - 5y = 10$

x	y
0	-2
5	0

if $x = 0$, $-5y = 10$, $y = -2$

if $y = 0$, $2x = 10$, $x = 5$



Test point: $(0,0) \notin$ line

$2(0) - 5(0) > 10$?

$0 > 10$ false

$\Rightarrow (0,0)$ not a solution

(6) (a) $ax + by = c$

$2x - 5y = 7$, $a = 2$, $b = -5$, $c = 7$

(b) $y = mx + b$ where m = slope, $(0,b)$ = y-int

$y = \frac{2}{3}x + 1$ where $m = \frac{2}{3}$, $(0,1)$ = y-int

(c) $y - y_1 = m(x - x_1)$ where m = slope, (x_1, y_1) = point on the line

$y - 3 = -(x + 2)$ where $m = -1$, $(-2, 3)$ = point

(d) $l_1 \parallel l_2$ iff $m_1 = m_2$

$y = 2x + 5$ and $y = 2x - 3$

$m_1 = m_2 = 2$

$b_1 = 5$, $b_2 = -3$ which shows the lines are distinct

(e) $l_1 \perp l_2$ iff $m_1 \cdot m_2 = -1$ (or $m_1 = \frac{-1}{m_2}$)

$y = \frac{4}{5}x + 1$ and $y = \frac{-5}{4}x + 3$

$m_1 = \frac{4}{5}$

$m_2 = -\frac{5}{4}$ and $m_1 m_2 = -1$

(f) $m = \frac{\Delta y}{\Delta x}$

$m = \frac{y_1 - y_2}{x_1 - x_2}$

where (x_1, y_1) and (x_2, y_2) are points on the line

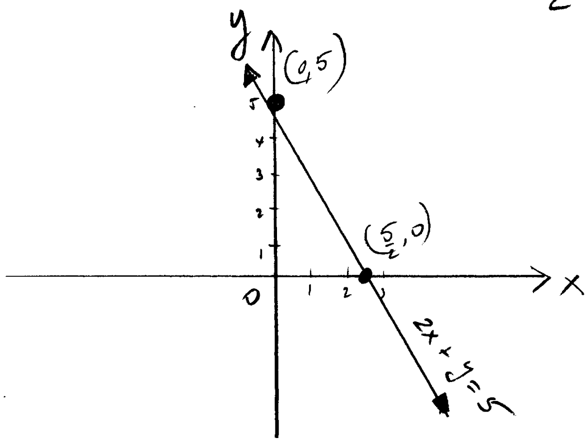
(7) (a) $2x + y = 5$

x	y
0	5
$\frac{5}{2}$	0

if $x = 0$, $y = 5$

if $y = 0$, $2x = 5$

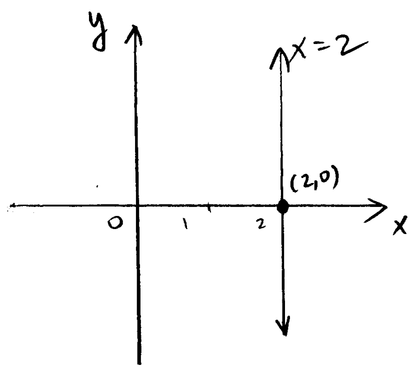
$x = \frac{5}{2}$



(b) $x - 2 = 0$

$x = 2$

vertical line

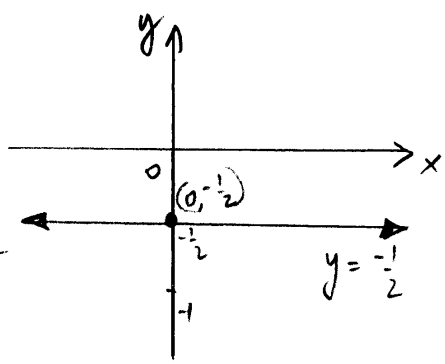


(c) $2y + 1 = 0$

$2y = -1$

$y = \frac{-1}{2}$

horizontal line



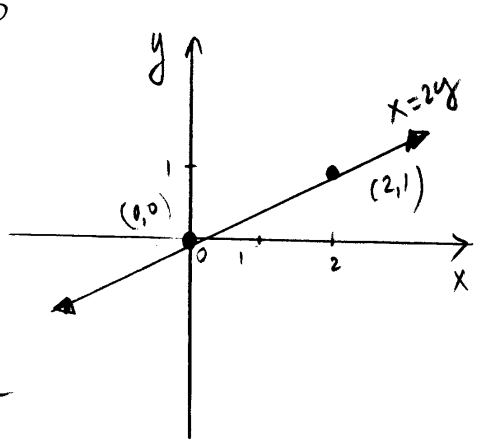
(d) $x - 2y = 0$

$x = 2y$

x	y
0	0
2	1

if $x = 0, y = 0$

if $y = 1, x = 2$



(8) $5x + 2y = 16$

(a) $2y = -5x + 16 \quad | \div 2$

$y = \frac{-5}{2}x + 8$

$\boxed{m = \frac{-5}{2}}$

(b) $m_{\parallel} = \frac{-5}{2}$

(c) $m_{\perp} = \frac{2}{5}$

(d) x-int: let $y = 0$

$5x = 16$

$x = \frac{16}{5}$

$\boxed{\text{x-int: } (\frac{16}{5}, 0)}$

y-int: let $x = 0$

$2y = 16$

$y = 8$

$\boxed{\text{y-int: } (0, 8)}$

(9) (a) $m = \frac{-2}{3}$

y-int: $(0, 3) \Rightarrow b = 3$

$y = mx + b$

$\boxed{y = \frac{-2}{3}x + 3}$ slope-int form

$\frac{2}{3}x + y = 3 \quad | \cdot 3$

$\boxed{2x + 3y = 9}$ standard form with integer coefficients

(b) $m=5$, $(-1,3)$

$y-y_1 = m(x-x_1)$

$y-3 = 5(x-(-1))$

$\boxed{y-3 = 5(x+1)}$ / slope-point form

$y-3 = 5x+5$

$y = 5x+5+3$

$\boxed{y = 5x+8}$ / slope-n form

(c) $(3,-1)$ and $(2,5)$

we need the slope m

$m = \frac{\Delta y}{\Delta x} = \frac{5-(-1)}{2-3} = \frac{6}{-1}$

$m = -6$

$y-y_1 = m(x-x_1)$ with $m=-6$, $(2,5)$

$\boxed{y-5 = -6(x-2)}$

(10) 40% acid, 70% acid, 50% acid

$\boxed{\text{20 liters}} + \boxed{\text{x liters}} = \boxed{\text{20+x liters}}$

let x = the number of liters of the 70% acid sol.

$40\%(20) + 70\%(x) = 50\%(20+x)$

$\frac{40}{100} \cdot 20 + \frac{70}{100}x = \frac{50}{100}(20+x)$ / $\cdot 100$

$40(20) + 70x = 50(20+x)$

$800 + 70x = 1000 + 50x$

$70x - 50x = 1000 - 800$

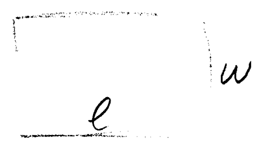
$20x = 200$

$x = \frac{200}{20}$

$x = 10$ liters

He needs to mix 10 liters of the 70% acid sol.

(11)



Given: perimeter = 120 ft

let l = length
w = width

$\begin{cases} 2l+2w=120 \\ l = 6+2w \end{cases}$ / $\div 2$

$\begin{cases} l+w=60 \\ l = 6+2w \end{cases}$

Substitution method.

$(6+2w)+w = 60$

$6+3w = 60$

$3w = 60-6$

$3w = 54$

$w = \frac{54}{3}$

$\boxed{w = 18 \text{ ft}}$ / width

then $l = 6+2w$

$l = 6+2(18)$

$\boxed{l = 42 \text{ ft}}$ / length

(12) $y = 4x - 1 \Rightarrow m_1 = 4$

$y - 3 = \frac{-1}{4}(x+1) \Rightarrow m_2 = \frac{-1}{4}$

$m_1 \cdot m_2 = 4\left(\frac{-1}{4}\right) = -1$

Therefor the lines are perpendicular.

(13) $\begin{cases} x + 4y = -8 \\ 3x + 2y = 6 \end{cases}$

(a) Graphical method

$x + 4y = -8$

x	y
0	-2
-8	0

if $x = 0$, $4y = -8$, $y = -2$

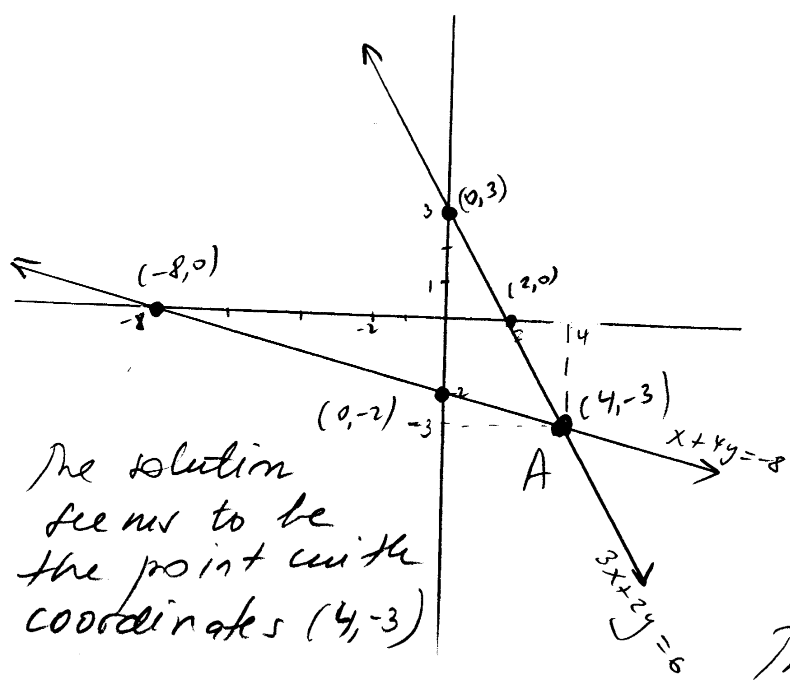
if $y = 0$, $x = -8$

$3x + 2y = 6$

x	y
0	3
2	0

if $x = 0$, $2y = 6$, $y = 3$

if $y = 0$, $3x = 6$, $x = 2$



The solution seems to be the point with coordinates $(4,-3)$

(b) Substitution method

$\begin{cases} x + 4y = -8 \Rightarrow x = -8 - 4y \\ 3x + 2y = 6 \end{cases}$

$3(-8 - 4y) + 2y = 6$

$-24 - 12y + 2y = 6$

$-24 - 10y = 6$

$-24 - 6 = +10y$

$-30 = +10y \Rightarrow y = -3$

$x = -8 - 4y$

$x = -8 - 4(-3)$

$x = -8 + 12 \Rightarrow x = 4$

The solution is $(4,-3)$

(c) Elimination method

$\begin{cases} x + 4y = -8 \\ 3x + 2y = 6 \end{cases} \Big| -3$

Eliminate x

$\begin{cases} -3x - 12y = 24 \\ 3x + 2y = 6 \end{cases}$

$(+)\quad -10y = 30$

$y = -3$

$x + 4y = -8$

$x + 4(-3) = -8$

$x - 12 = -8$

$x = 12 - 8$

$x = 4$

The solution is $(4,-3)$

(14) (a) t = time (in minutes)
B = number of barrels (in thousands)

t = independent variable
B = dependent variable

(b) (10, 750)
when t = 10, B = 750
After 10 minutes, there are 750 thousand barrels of oil left in the tanker

(c) let A (0, 800)
B (10, 750)
C (20, 700)
D (30, 650)

$m_{AB} = \frac{\Delta B}{\Delta t} = \frac{800-750}{0-10} = -5$

$m_{AC} = \frac{\Delta B}{\Delta t} = \frac{800-700}{0-20} = -5$

$m_{AD} = \frac{\Delta B}{\Delta t} = \frac{800-650}{0-30} = -5$

Therefore, A, B, C, D are collinear
yes, the table shows a linear equation

(d)
m = -5 thousand barrels / min

The slope shows the rate at which the amount of oil in the tanker decreases
The oil in the tanker decreases by 5000 barrels per minute.