

QUIZ #3 @ 85 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. For an exercise to be complete there needs to be a detailed solution to the problem. No proof, no credit given! Clearly label each exercise.

1. Do the following operations and simplify:

a) $\frac{1+t^3}{1+t}$

e) $4\sqrt{50} + 3\sqrt{12} - 5\sqrt{45}$

i) $(5\sqrt{7} - 2\sqrt{3})^2$

b) $\frac{3}{m+1} - \frac{4}{m}$

f) $(5\sqrt{7} - 2\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$

j) $(\sqrt[3]{2} - 1)(\sqrt[3]{4} + 3)$

c) $2\sqrt{12} + 3\sqrt{75}$

g) $(\sqrt{6} + 1)^2$

k) $(\sqrt{x} + \sqrt{3x-1})^2$

d) $2\sqrt[3]{32m^3} - \sqrt[3]{108m^3}$

h) $\frac{16 + 8\sqrt{2}}{24}$

l) $\frac{12 - 4\sqrt{10}}{4}$

2. Solve the following equations.

a) $\frac{1}{x-1} + \frac{1}{2} = \frac{2}{x^2-1}$

d) $5\sqrt{x} = \sqrt{10x+15}$

f) $\sqrt{2x+11} + \sqrt{x+6} = 2$

b) $\frac{x}{4-x} = \frac{2}{x}$

e) $x = \sqrt{x^2 - 2x - 6}$

g) $\sqrt{5x+11} - x = 3$

c) $\frac{3}{x+3} + \frac{4}{x+6} = \frac{9}{x^2+9x+18}$

3) Rationalize each denominator and simplify.

a) $\frac{2}{\sqrt{3}}$

b) $\frac{5}{\sqrt[3]{2}}$

c) $\frac{1}{4 + \sqrt{15}}$

3. Vince can fly his plane 200 mi against the wind in the same time it takes him to fly 300 mi with the wind. Find the speed of his plane in still air if the wind blows at 30 mph.

4. Mark can tune up his car in 2 hours working alone. Ben can do the job in 3 hours working alone. How long would it take them if they worked together?

5. Find the distance between the points $(-1, 3)$ and $(5, -2)$.

Quiz 3 - Solutions

$$\textcircled{1} \textcircled{a} \frac{1+t^3}{1+t} = \frac{(1+t)(1-t+t^2)}{1+t} = \boxed{1-t+t^2}$$

$$\textcircled{b} \frac{3}{m+1} - \frac{4}{m} = \frac{3m-4(m+1)}{m(m+1)}$$

$$\text{LCO} = m(m+1)$$

$$= \frac{3m-4m-4}{m(m+1)} = \boxed{\frac{-m-4}{m(m+1)}}$$

$$\textcircled{c} 2\sqrt{12} + 3\sqrt{75} = 2\sqrt{4 \cdot 3} + 3\sqrt{25 \cdot 3}$$

$$= 2 \cdot 2\sqrt{3} + 3 \cdot 5\sqrt{3}$$

$$= 4\sqrt{3} + 15\sqrt{3} = \boxed{19\sqrt{3}}$$

$$\textcircled{d} 2\sqrt[3]{32m^3} - \sqrt[3]{108m^3} =$$

$$= 2\sqrt[3]{8 \cdot 4m^3} - \sqrt[3]{27 \cdot 4m^3}$$

$$= 2 \cdot 2m\sqrt[3]{4} - 3m\sqrt[3]{4} = \boxed{m\sqrt[3]{4}}$$

$$\textcircled{e} 4\sqrt{50} + 3\sqrt{12} - 5\sqrt{45} =$$

$$= 4\sqrt{25 \cdot 2} + 3\sqrt{4 \cdot 3} - 5\sqrt{9 \cdot 5}$$

$$= 4 \cdot 5\sqrt{2} + 3 \cdot 2\sqrt{3} - 5 \cdot 3\sqrt{5}$$

$$= \boxed{20\sqrt{2} + 6\sqrt{3} - 15\sqrt{5}}$$

$$\textcircled{f} (5\sqrt{7} - 2\sqrt{3})(3\sqrt{7} + 4\sqrt{3}) =$$

$$= 15(\sqrt{7})^2 + 20\sqrt{7} \cdot \sqrt{3} - 6\sqrt{3} \cdot \sqrt{7} - 8(\sqrt{3})^2$$

$$= 15 \cdot 7 + 20\sqrt{21} - 6\sqrt{21} - 8 \cdot 3$$

$$= \boxed{81 + 14\sqrt{21}}$$

$$\textcircled{g} (\sqrt{6} + 1)^2 = (\sqrt{6})^2 + 2\sqrt{6} + 1^2$$

$$= 6 + 2\sqrt{6} + 1 = \boxed{7 + 2\sqrt{6}}$$

$$\textcircled{h} \frac{16 + 8\sqrt{2}}{24} = \frac{8(2 + \sqrt{2})}{24} = \boxed{\frac{2 + \sqrt{2}}{3}}$$

$$\textcircled{1} (5\sqrt{7} - 2\sqrt{3})^2 =$$

$$= (5\sqrt{7})^2 - 2(5\sqrt{7})(2\sqrt{3}) + (2\sqrt{3})^2$$

$$= 25 \cdot 7 - 20 \cdot \sqrt{21} + 4 \cdot 3 = \boxed{187 - 20\sqrt{21}}$$

$$\textcircled{1} (\sqrt[3]{2} - 1)(\sqrt[3]{4} + 3) =$$

$$= \sqrt[3]{2} \cdot \sqrt[3]{4} + 3\sqrt[3]{2} - \sqrt[3]{4} - 3$$

$$= \sqrt[3]{8} + 3\sqrt[3]{2} - \sqrt[3]{4} - 3$$

$$= 2 + 3\sqrt[3]{2} - \sqrt[3]{4} - 3 = \boxed{3\sqrt[3]{2} - \sqrt[3]{4} - 1}$$

$$\textcircled{k} (\sqrt{x} + \sqrt{3x-1})^2 =$$

$$= (\sqrt{x})^2 + 2\sqrt{x} \sqrt{3x-1} + (\sqrt{3x-1})^2$$

$$= x + 2\sqrt{x(3x-1)} + 3x-1$$

$$= \boxed{4x-1 + 2\sqrt{x(3x-1)}}$$

$$\textcircled{l} \frac{12 - 4\sqrt{10}}{4} = \frac{4(3 - \sqrt{10})}{4} = \boxed{3 - \sqrt{10}}$$

$$\textcircled{2} \textcircled{a} \frac{1}{x-1} + \frac{1}{2} = \frac{2}{x^2-1}$$

$$\frac{2(x+1)}{2(x+1)(x-1)} + \frac{1}{2} = \frac{2}{(x-1)(x+1)}$$

Conditions: $\begin{cases} x \neq 1 \\ x \neq -1 \end{cases}$ LCO = $2(x-1)(x+1)$

$$2(x+1) + (x+1)(x-1) = 4$$

$$2x+2 + x^2-1-4=0$$

$$x^2+2x-3=0$$

$$(x+3)(x-1) = 0$$

$$x+3=0 \text{ OR } x-1=0$$

$$x=-3$$

$x=1$ not possible

$$\boxed{x \in \{-3\}}$$

-3-

(9) $\sqrt{5x+11} - x = 3$

$\sqrt{5x+11} = 3+x$

$(\sqrt{5x+11})^2 = (3+x)^2$

$5x+11 = 9+6x+x^2$

$9+6x+x^2 - 5x - 11 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x+2=0$ OR $x-1=0$

$x=-2$ OR $x=1$

check $x=-2$

$\sqrt{5(-2)+11} - (-2) \stackrel{?}{=} 3$

$1+2=3$ TRUE

check $x=1$

$\sqrt{5(1)+11} - 1 \stackrel{?}{=} 3$

$4-1=3$ TRUE

True for, $x \in \{-2, 1\}$

	distance	rate	time
(7) against wind	200 mi	$x-30$ mph	same time
with wind	300 mi	$x+30$ mph	

let $x =$ speed of plane in still air

time against wind = time with wind

we know time = $\frac{\text{distance}}{\text{rate}}$

$\frac{200}{x-30} = \frac{300}{x+30}$

$\frac{2}{x-30} = \frac{3}{x+30}$

Apply cross-product property

$2(x+30) = 3(x-30)$

$2x+60 = 3x-90$

$60+90 = 3x-2x$

$150 = x$

$x = 150$ mph - the speed of the plane in still air

(3) (a) $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{(\sqrt{3})^2} = \frac{2\sqrt{3}}{3}$

(b) $\frac{5}{\sqrt[3]{2}} = \frac{5\sqrt[3]{4}}{\sqrt[3]{2}\sqrt[3]{4}} = \frac{5\sqrt[3]{4}}{2}$

(c) $\frac{4-\sqrt{15}}{4+\sqrt{15}} = \frac{4-\sqrt{15}}{(4+\sqrt{15})(4-\sqrt{15})}$

$= \frac{4-\sqrt{15}}{4^2 - (\sqrt{15})^2}$

$= \frac{4-\sqrt{15}}{16-15} = 4-\sqrt{15}$

	Time to do the job	Part of job done per hour
(5) Mark	2 hr	$\frac{1}{2}$
Ken	3 hr	$\frac{1}{3}$
together	x hr	$\frac{1}{x}$

let $x =$ the number of hours it takes them to finish the job together

$\frac{1}{2} + \frac{1}{3} = \frac{1}{x}$

$\frac{3+2}{6} = \frac{1}{x}$

$\frac{5}{6} = \frac{1}{x} \Rightarrow 5x = 6$

$x = \frac{6}{5}$ hrs

(b) $\frac{x}{4-x} = \frac{2}{x}$

Conditions: $\begin{cases} x \neq 4 \\ x \neq 0 \end{cases}$

Apply cross-product property:

$x^2 = 2(4-x)$

$x^2 = 8 - 2x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x+4=0$ OR $x-2=0$

$x=-4$

$x=2$

$x \in \{-4, 2\}$

(c) $\frac{3}{x+3} + \frac{4}{x+6} = \frac{9}{x^2+9x+18}$

$\frac{3}{x+3} + \frac{4}{x+6} = \frac{9}{(x+3)(x+6)}$

Conditions: $\begin{cases} x \neq -3 \\ x \neq -6 \end{cases}$ LCO = $(x+3)(x+6)$

$3(x+6) + 4(x+3) = 9$

$3x+18 + 4x+12 = 9$

$7x+30 = 9$

$7x = 9-30$

$7x = -21 \Rightarrow x = -3$ not possible

$x \in \emptyset$

(d) $5\sqrt{x} = \sqrt{10x+15}$

$(5\sqrt{x})^2 = (\sqrt{10x+15})^2$

$25x = 10x+15$

$25x - 10x = 15$

$15x = 15$

$x=1$

Check:
 $5\sqrt{1} \stackrel{?}{=} \sqrt{10 \cdot 1 + 15}$
 $5 = 5$ TRUE

Therefore,
 $x \in \{1\}$

(e) $x = \sqrt{x^2-2x-6}$

$x^2 = (\sqrt{x^2-2x-6})^2$

$x^2 = x^2 - 2x - 6$

$0 = -2x - 6$

$2x = -6$

$x = -3$

Check

$-3 = \sqrt{(-3)^2 - 2(-3) - 6}$

not possible

Therefore, $x \in \emptyset$

(f) $\sqrt{2x+11} + \sqrt{x+6} = 2$

$\sqrt{2x+11} = 2 - \sqrt{x+6}$

$(\sqrt{2x+11})^2 = (2 - \sqrt{x+6})^2$

$2x+11 = 4 - 4\sqrt{x+6} + x+6$

$2x+11 = 10 - 4\sqrt{x+6} + x$

$2x+11 - 10 - x = -4\sqrt{x+6}$

$x+1 = -4\sqrt{x+6}$

$(x+1)^2 = (-4\sqrt{x+6})^2$

$x^2 + 2x + 1 = 16(x+6)$

$x^2 + 2x + 1 = 16x + 96$

$x^2 + 2x + 1 - 16x - 96 = 0$

$x^2 - 14x - 95 = 0$

product = $-95 \begin{matrix} +5 \\ -19 \end{matrix}$
sum = -14

$95 = 5 \cdot 19$

$(x+5)(x-19) = 0$

$x+5=0$ OR $x-19=0$

$x=-5$

$x=19$

Check $x=-5$

$\sqrt{2(-5)+11} + \sqrt{-5+6} \stackrel{?}{=} 2$

$1+1 = 2$ TRUE

Check $x=19$

$\sqrt{2 \cdot 19 + 11} + \sqrt{19+6} \stackrel{?}{=} 2$ FALSE

Therefore, $x \in \{-5\}$

$$\textcircled{6} \quad d^2 = (\Delta x)^2 + (\Delta y)^2$$

$$d^2 = (5-(-1))^2 + (3-(-2))^2$$

$$d^2 = 6^2 + 5^2$$

$$d^2 = 36 + 25$$

$$d^2 = 61$$

$$d = \sqrt{61} \quad \text{or} \quad d = \cancel{\sqrt{61}}$$

not possible

The distance is $\sqrt{61}$.