

QUIZ #2 @ 85 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. Use a #2 pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write an answer. No proof, no credit given! Clearly label each exercise.

1. Solve the following system by the substitution method:

$$\begin{cases} \frac{x}{5} + y = \frac{6}{5} \\ \frac{x}{10} + \frac{y}{3} = \frac{5}{6} \end{cases}$$

2. Solve the following system by the elimination method:

$$\begin{cases} 9x + 4y = -3 \\ 6x + 6y = -7 \end{cases}$$

3. Write a system of equation for the problem, and then solve the system. Clearly define the variables.
 A 90% antifreeze solution is to be mixed with a 75% solution to make 120 L of a 78% solution. How many liters of the 90% and 75% solutions will be used?

4. Simplify the following g expressions (if possible). Write the final answer using only positive exponents (if necessary).

| | | |
|------------------------|---------------------------------------------------------------|-----------------------------------------------------------------|
| a) $a^{-2} + 2a^{-2}$ | e) $\frac{(2x^{-1}y)^2}{x^{-3}y^5}$ | i) $\left(9y + \frac{2}{3}\right)\left(9y - \frac{2}{3}\right)$ |
| b) $x^2 - 2x^3$ | f) $3^{-2} + \left(\frac{1}{2}\right)^{-1} + (-2)^{-1} + 5^0$ | j) $q(5q-1)(8q+7)$ |
| c) $t^{-1} - t^{-1}$ | g) $(y^3 - 5)^2$ | |
| d) $(t^{-1})(-t^{-1})$ | h) $\left(\frac{1}{3}a - \frac{9}{2}c\right)^2$ | |

5. Do the following division: $\frac{x^4 - 4x^3 + 5x^2 - 3x + 2}{x^2 + 3}$

6. Factor each expression as completely as possible. If prime, state so.

| | |
|-----------------------|--------------------------|
| a) $2x + 6 + ax + 3a$ | d) $y^2 - 5y + 12$ |
| b) $3(5-x) + y(5-x)$ | e) $2x^6 + 8x^5 - 42x^4$ |
| c) $m^2 + 9m + 14$ | f) $d^2 + 4d - 45$ |

Quiz 2 - SOLUTIONS

$$\textcircled{1} \begin{cases} \frac{x}{5} + \frac{y}{5} = \frac{6}{5} \\ \frac{3x}{10} + \frac{y}{3} = \frac{5}{6} \end{cases} \begin{array}{l} LCO = 5 \\ LCO = 30 \end{array}$$

$$\begin{cases} x + 5y = 6 \Rightarrow x = 6 - 5y \\ 3x + 10y = 25 \end{cases}$$

$$3(6 - 5y) + 10y = 25$$

$$18 - 15y + 10y = 25$$

$$18 - 5y = 25$$

$$18 - 25 = 5y$$

$$-7 = 5y \Rightarrow y = -\frac{7}{5}$$

$$x = 6 - 5y$$

$$x = 6 - 5\left(-\frac{7}{5}\right)$$

$$x = 6 + 7 \Rightarrow x = 13$$

The solution is $\boxed{\left(13, -\frac{7}{5}\right)}$

$$\textcircled{2} \begin{cases} 9x + 4y = -3 \quad | \cdot 3 \\ 6x + 6y = -7 \quad | \cdot (-2) \end{cases}$$

Eliminate y ; $LCM(4, 6) = 12$

$$\begin{cases} 27x + 12y = -9 \\ -12x - 12y = 14 \end{cases}$$

$$\textcircled{+} 15x = 5 \Rightarrow x = \frac{5}{15} = \frac{1}{3}$$

$$9x + 4y = -3$$

$$9 \cdot \frac{1}{3} + 4y = -3$$

$$3 + 4y = -3 \Rightarrow 4y = -6 \Rightarrow y = -\frac{6}{4}$$

The solution is $\boxed{\left(\frac{1}{3}, -\frac{3}{2}\right)}$

$\textcircled{3}$ Let x = the number of liters of the 90% solution
 y = the number of liters of the 75% sol.

$$\begin{array}{c} 90\% \quad 75\% \quad 78\% \\ \boxed{xL} + \boxed{yL} = \boxed{120L} \end{array}$$

$$\begin{cases} x + y = 120 \\ 90\%x + 75\%y = 78\%(120) \end{cases}$$

$$\begin{cases} x + y = 120 \\ \frac{90}{100}x + \frac{75}{100}y = \frac{78}{100} \cdot 120 \end{cases} | \cdot (100)$$

$$\begin{cases} x + y = 120 \\ 90x + 75y = 78(120) \end{cases} | -90$$

Eliminate x :

$$\begin{cases} -90x - 90y = -10,800 \\ 90x + 75y = 9360 \end{cases}$$

$$\textcircled{+} -15y = -1440$$

$$y = \frac{1440}{15} = \frac{480}{5} = 96$$

$$x + y = 120$$

$$x + 96 = 120$$

$$x = 120 - 96 = 24$$

24 L of 90% solution
and
96 L of 75% solution

$$(4) (a) a^{-2} + 2a^{-2} = 3a^{-2} = \boxed{\frac{3}{a^2}}$$

$$(h) \left(\frac{1}{3}a - \frac{9}{2}c\right)^2 =$$

$$(b) x^2 - 2x^3 - \text{already simplified}$$

$$= \left(\frac{1}{3}a\right)^2 - 2 \cdot \frac{1}{3}a \cdot \frac{9}{2}c + \left(\frac{9}{2}c\right)^2$$

$$(c) t^{-1} - t^{-1} = 0$$

$$= \frac{1}{9}a^2 - 3ac + \frac{81}{4}c^2$$

$$(d) (t^{-1})(-t^{-1}) = -t^{-2} = -\frac{1}{t^2}$$

$$(i) \left(9y + \frac{2}{3}\right)\left(9y - \frac{2}{3}\right) =$$

$$= (9y)^2 - \left(\frac{2}{3}\right)^2$$

$$= \boxed{81y^2 - \frac{4}{9}}$$

$$(e) \frac{(2x^{-1}y)^2}{x^{-3}y^5} = \frac{2^2(x^{-1})^2y^2}{x^{-3}y^5}$$

$$= \frac{4x^{-2}y^2}{x^{-3}y^5} = 4x^{-2-(-3)}y^{2-5}$$

$$= 4xy^{-3} = \boxed{\frac{4x}{y^3}}$$

$$(j) 9(5q-1)(5q+1) =$$

$$= 9((5q)^2 - 1^2)$$

$$= 9(25q^2 - 1)$$

$$= \boxed{25q^3 - 9}$$

$$(f) 3^{-2} + \left(\frac{1}{2}\right)^{-1} + (-2)^{-1} + 5^0 =$$

$$= \frac{1}{3^2} + \frac{1}{\frac{1}{2}} + \frac{1}{-2} + 1$$

$$= \frac{1}{9} + 2 - \frac{1}{2} + 1$$

$$= \frac{2}{9} - \frac{9}{18} + \frac{12}{18} \quad \text{LCD} = 18$$

$$= \frac{2}{18} - \frac{9}{18} + \frac{54}{18} = \frac{2-9+54}{18}$$

$$= \boxed{\frac{47}{18}}$$

$$(g) (y^3-5)^2 = (y^3)^2 - 2 \cdot y^3 \cdot 5 + 5^2$$

$$= \boxed{y^6 - 10y^3 + 25}$$

$$(5) \begin{array}{r} x^2 - 4x + 2 \\ x^2 + 3 \sqrt{x^4 - 4x^3 + 5x^2 - 3x + 2} \\ \quad -x^4 \qquad \quad -3x^2 \\ \hline \quad \quad -4x^3 + 2x^2 - 3x + 2 \\ \quad \quad +4x^3 \qquad \quad +12x \\ \hline \quad \quad \quad 2x^2 + 9x + 2 \\ \quad \quad \quad -2x^2 \qquad \quad -6 \\ \hline \quad \quad \quad \quad 9x - 4 \end{array}$$

$$\frac{x^4 - 4x^3 + 5x^2 - 3x + 2}{x^2 + 3} =$$

$$= \boxed{x^2 - 4x + 2 + \frac{9x - 4}{x^2 + 3}}$$

-3-

(6)

$$\begin{aligned} \text{(a)} \quad & 2x+6+ax+3a = \\ & = 2(x+3)+a(x+3) \\ & = \boxed{(x+3)(2+a)} \end{aligned}$$

(7)

$$\begin{aligned} d^2+4d-45 &= \\ &= \boxed{(d+9)(d-5)} \end{aligned}$$

product = c = -45 $\begin{matrix} +9 \\ -5 \end{matrix}$
 sum = b = 4
 45 = 9 · 5

$$\begin{aligned} \text{(b)} \quad & 3(5-x)+y(5-x) = \\ & = \boxed{(5-x)(3+y)} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & m^2+9m+14 = \\ & = \boxed{(m+7)(m+2)} \end{aligned}$$

product = c = 14 $\begin{matrix} +7 \\ +2 \end{matrix}$
 sum = b = 9
 14 = 7 · 2

$$\begin{aligned} \text{(d)} \quad & y^2-5y+12 = \boxed{\text{prime}} \\ & \text{product} = c = 12 < - \\ & \text{sum} = b = -5 \\ & 12 = 1 \cdot 12 \\ & \quad = 6 \cdot 2 \\ & \quad = 4 \cdot 3 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad & 2x^6+8x^5-42x^4 = \\ & = 2x^4(x^2+4x-21) \\ & = \boxed{2x^4(x+7)(x-3)} \end{aligned}$$

product = c = -21 $\begin{matrix} +7 \\ -3 \end{matrix}$
 sum = b = 4
 21 = 7 · 3