Math 180 Spring 2009 Name:

QUIZ #2 @ 50 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1. Find the derivative of each function:

a) $y = \sqrt[5]{x^2}$

b) $s(t) = \dfrac{t^2 + 5t - 1}{t^2 + 1}$

c) $y = 2x^3 e^x - \dfrac{1}{x}$

d) $r = \dfrac{\cos\theta}{1 - \sin\theta}$

e) $y = \sqrt{3x + x^2}$

f) $y = \sin\left(\cos(2t - 5)\right)$

g) $f(x) = \tan\left(x^2 + e^x\right)$

h) $y = t(\ln t)^2$

i) $y = 2^x + \log_3\left(x^2\right)$

2. Use implicit differentiation to find dy/dx if

$$x^2 y + xy^2 = 6.$$

3. Use logarithmic differentiation to find dy/dx if

$$y = \frac{\left(x^2 + 1\right)\sqrt{x + 3}}{x - 1}, \qquad x > 1.$$

SOLUTIONS

(1) (a) $y = \sqrt[5]{x^2} = x^{\frac{2}{5}}$

$y' = \left(x^{\frac{2}{5}}\right)' = \frac{2}{5} x^{\frac{2}{5}-1}$

$= \frac{2}{5} x^{-\frac{3}{5}}$

$y' = \frac{2}{5\sqrt[5]{x^3}}$

(b) $s(t) = \frac{t^2+5t-1}{t^2+1}$

$s'(t) = \frac{ds}{dt} = \frac{(t^2+5t-1)'(t^2+1)-(t^2+5t-1)(t^2+1)'}{(t^2+1)^2}$

$= \frac{(2t+5)(t^2+1)-(t^2+5t-1)(2t)}{(t^2+1)^2}$

$= \frac{\cancel{2t^3}+2t+5t^2+5-\cancel{2t^3}-10t^2+2t}{(t^2+1)^2}$

$s' = \frac{-5t^2+4t+5}{(t^2+1)^2}$

(c) $y = 2x^3e^x - \frac{1}{x}$

$y = 2x^3e^x - x^{-1}$

$y' = 2\left[(x^3)'e^x + x^3(e^x)'\right] - (x^{-1})'$

$= 2(3x^2)e^x + 2x^3e^x - (-1)x^{-2}$

$y' = 6x^2e^x + 2x^3e^x + x^{-2}$

(d) $r = \frac{\cos\theta}{1-\sin\theta}$

$r' = \frac{dr}{d\theta}$

$= \frac{(\cos\theta)'(1-\sin\theta)-\cos\theta(1-\sin\theta)'}{(1-\sin\theta)^2}$

$= \frac{-\sin\theta(1-\sin\theta)-\cos\theta(-\cos\theta)}{(1-\sin\theta)^2}$

$= \frac{-\sin\theta+\sin^2\theta+\cos^2\theta}{(1-\sin\theta)^2}$

$= \frac{1-\sin\theta}{(1-\sin\theta)^2} = \frac{1}{1-\sin\theta}$

$r' = \frac{1}{1-\sin\theta}$

(e) $y = \sqrt{3x+x^2}$

$y' = \frac{dy}{dx} = \frac{1}{2\sqrt{3x+x^2}}(3x+x^2)'$

$y' = \frac{3+2x}{2\sqrt{3x+x^2}}$

(7) $y = \sin(\cos(2t-5))$

$y' = \cos(\cos(2t-5)) \cdot (\cos(2t-5))'$

$= \cos(\cos(2t-5))(-\sin(2t-5)) \cdot (2t-5)'$

$y' = -\cos(\cos(2t-5))\sin(2t-5) \cdot 2$

$y' = -2\cos(\cos(2t-5))\sin(2t-5)$

(g) $f(x) = \tan(x^2+e^x)$

$f'(x) = \sec^2(x^2+e^x)\cdot(x^2+e^x)'$

$f'(x) = (2x+e^x)\sec^2(x^2+e^x)$

(h) $y = t(\ln t)^2$

$y' = t'(\ln t)^2 + t((\ln t)^2)'$

$= \ln^2 t + t\cdot 2\ln t\cdot(\ln t)'$

$= \ln^2 t + 2\cancel{t}\ln t\cdot\frac{1}{\cancel{t}}$

$y' = \ln^2 t + 2\ln t$

(i) $y = 2^x + \log_3(x^2)$

$y' = 2^x\ln 2 + \frac{1}{x^2\ln 3}\cdot(x^2)'$

$y' = 2^x\ln 2 + \frac{2x}{x^2\ln 3}$

$y' = 2^x\ln 2 + \frac{2}{x\ln 3}$

(2) $x^2y + xy^2 = 6 \quad \Big/ \cdot \frac{d}{dx}$

$2xy + x^2y' + y^2 + x(2y)y' = 0$

$x^2y' + 2xyy' = -2xy - y^2$

$y'(x^2+2xy) = -2xy - y^2$

$y' = \frac{-2xy-y^2}{x^2+2xy}$

(3) $y = \frac{(x^2+1)\sqrt{x+3}}{x-1}$

$\ln y = \ln\frac{(x^2+1)\sqrt{x+3}}{x-1}$

$\ln y = \ln(x^2+1) + \frac{1}{2}\ln(x+3) -$
$- \ln(x-1)$

$\frac{1}{y}\cdot y' = \frac{2x}{x^2+1} + \frac{1}{2(x+3)} - \frac{1}{x-1}$

$y' = y\left(\frac{2x}{x^2+1} + \frac{1}{2(x+3)} - \frac{1}{x-1}\right)$

$y' = \frac{(x^2+1)\sqrt{x+3}}{x-1}\left(\frac{2x}{x^2+1} + \frac{1}{2(x+3)} - \frac{1}{x-1}\right)$