Math 71 – Spring 2008 Name:

QUIZ #3 @ 50 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Solve by extracting roots in the set of complex numbers ($\mathbb{C}$): $2(x+3)^2 - 48 = 0$

2. Solve by completing the square in the set of complex numbers ($\mathbb{C}$): $2x^2 + 3x - 4 = 0$

3. Solve by the quadratic formula in the set of complex numbers ($\mathbb{C}$): $9x^2 + x = -2$

4. Solve the following equations in the set of complex numbers ($\mathbb{C}$):

a) $4a = 16a^3$

b) $100 - 3(x-1)^2 = 1$

c) $2x^2 = -4x - 5$

d) $4x^3 - x^2 - 12x + 3 = 0$

5. $f(x) = -x^2 - 2x + 1$

a) Graph the function. Show all work. Label all points and the axes.

b) State the domain and range.

c) Using the graph, solve the following inequality: $-x^2 - 2x + 1 \le 0$

d) Write the equation in vertex form.

Math 71

Quiz #3 - Solutions

① $2(x+3)^2 - 48 = 0$

$2(x+3)^2 = 48 \quad / \div 2$

$(x+3)^2 = 24 \quad / \sqrt{}$

$\sqrt{(x+3)^2} = \sqrt{24}$

$x+3 = \pm\sqrt{24}$

$x = -3 \pm 2\sqrt{6}$

$\boxed{x \in \{-3 \pm 2\sqrt{6}\}}$ /

② $2x^2 + 3x - 4 = 0$

$2x^2 + 3x = 4 \quad / \div 2$

$x^2 + \frac{3}{2}x = 2 \quad / + \frac{9}{16}$

$\left(\frac{1}{2}\text{coef}\cdot x\right)^2 = \left(\frac{1}{2}\cdot\frac{3}{2}\right)^2 = \frac{9}{16}$

$x^2 + \frac{3}{2}x + \frac{9}{16} = 2 + \frac{9}{16}$

$\left(x + \frac{3}{4}\right)^2 = \frac{41}{16} \quad / \sqrt{}$

$\sqrt{\left(x+\frac{3}{4}\right)^2} = \sqrt{\frac{41}{16}}$

$x + \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$

$x = -\frac{3}{4} \pm \frac{\sqrt{41}}{4}$

$\boxed{x \in \left\{-\frac{3}{4} \pm \frac{\sqrt{41}}{4}\right\}}$ /

③ $9x^2 + x = -2$

$9x^2 + x + 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \begin{cases} a = 9 \\ b = 1 \\ c = 2 \end{cases}$

$x = \frac{-1 \pm \sqrt{1 - 4(9)(2)}}{2(9)}$

$x = \frac{-1 \pm \sqrt{1-72}}{18} = \frac{-1 \pm \sqrt{-71}}{18}$

$x = \frac{-1 \pm i\sqrt{71}}{18}$

$\boxed{x \in \left\{\frac{-1 \pm i\sqrt{71}}{18}\right\}}$ /

④ ⓐ $4a = 16a^3$

$16a^3 - 4a = 0$

$4a(4a^2 - 1) = 0$

$4a(2a-1)(2a+1) = 0$

$a = 0$ OR

$2a - 1 = 0, \quad a = \frac{1}{2}$ OR

$2a + 1 = 0, \quad a = -\frac{1}{2}$

$\boxed{a \in \left\{0, \pm\frac{1}{2}\right\}}$ /

ⓑ $100 - 3(x-1)^2 = 1$

$100 - 1 = 3(x-1)^2$

$99 = 3(x-1)^2 \quad / \div 3$

$(x-1)^2 = 33 \quad / \sqrt{}$

$\sqrt{(x-1)^2} = \sqrt{33}$

$x - 1 = \pm\sqrt{33}$

$x = 1 \pm \sqrt{33}$

$\boxed{x \in \{1 \pm \sqrt{33}\}}$ /

(c) $2x^2 = -4x - 5$

$2x^2 + 4x + 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\left\{ \begin{array}{l} a = 2 \\ b = 4 \\ c = 5 \end{array} \right.$

$x = \dfrac{-4 \pm \sqrt{16 - 4(2)(5)}}{2(2)}$

$x = \dfrac{-4 \pm \sqrt{16 - 40}}{4} = \dfrac{-4 \pm \sqrt{-24}}{4}$

$x = \dfrac{-4 \pm 2i\sqrt{6}}{4} = \dfrac{2(-2 \pm i\sqrt{6})}{4}$

$x = \dfrac{-2 \pm i\sqrt{6}}{2}$

$\boxed{x \in \left\{ \dfrac{-2 \pm i\sqrt{6}}{2} \right\}}$ /

(d) $\underbrace{4x^3 - x^2}\underbrace{- 12x + 3} = 0$

$x^2(4x-1) - 3(4x-1) = 0$

$(4x-1)(x^2-3) = 0$

$4x - 1 = 0$ OR $x^2 - 3 = 0$ //

$4x = 1$ $\quad x^2 = 3$

$x = \frac{1}{4}$ $\quad x = \pm\sqrt{3}$

$\boxed{x \in \left\{ \frac{1}{4}, \pm\sqrt{3} \right\}}$

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(5) $f(x) = -x^2 - 2x + 1$

(a) parabola opening down $(a = -1 < 0)$

$V(x_v, y_v)$ $\quad x_v = \dfrac{-b}{2a} = \dfrac{-(-2)}{2(-1)} = -1$

$y_v = -(-1)^2 - 2(-1) + 1$

$y_v = -1 + 2 + 1 = 2$

$\boxed{V(-1, 2)}$ /

y-n: $x = 0$, $y = 1$

$\boxed{\text{y-n: } (0,1)}$ /

x-n: $y = 0$, $-x^2 - 2x + 1 = 0$

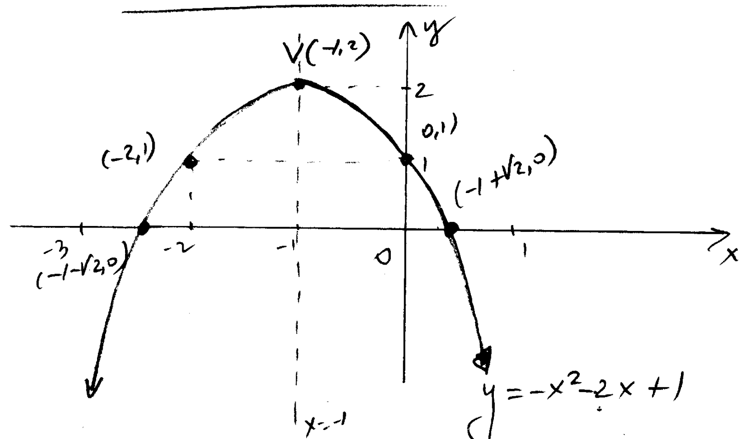
$x^2 + 2x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \dfrac{-2 \pm \sqrt{8}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2} \begin{cases} \approx 0.4 \\ \approx -2.4 \end{cases}$

$\boxed{\text{x-n: } (-1 \pm \sqrt{2}, 0)}$ /



(b) Domain: $x \in \mathbb{R}$

Range: $y \in (-\infty, 2]$

(c) $-x^2 - 2x + 1 \le 0$

iff

$x \in (-\infty, -1-\sqrt{2}] \cup [-1+\sqrt{2}, \infty)$

(d) $y = a(x - x_v)^2 + y_v$

$\boxed{y = -(x+1)^2 + 2}$