Math 150 Spring 08 Name:

Quiz #2 @ 85 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

1. Graph the function $y = \cos x$. Show the graph over two periods. Answer the following questions:
   a) What is the domain?
   b) What is the range?
   c) What is the period?
   d) What is the amplitude?
   e) What are the x-intercepts?
   f) What is the y-intercept?
   g) Is the function even or odd? How is that shown in the graph?

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2. Graph the following functions on graphing paper. In each case, identify the amplitude (when defined), the period, and phase shift (when defined) and label the axes accurately. Explain in words what and how you are graphing.

a) $y = 1 + \sin x$ from -2π to 4π

b) $y = -2\sin x$ over one period

c) $y = 3\cos\frac{1}{3}x$ over one period

d) $y = 3\sin 2\left(x - \frac{\pi}{3}\right)$ over one period

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3. Find all real numbers x that satisfy each equation. Justify your answers.

a) $\cos x = 0$

b) $\sin x = 0$

c) $\tan x = 0$

d) $\cot x = 0$

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4. Evaluate the following. Give exact answers whenever possible. Justify your answer.

a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

c) $\sin^{-1}(\sin(0.4))$

d) $\cos\left(\sin^{-1}\frac{3}{5}\right)$

e) $\sin^{-1}\left(\sin\frac{5\pi}{8}\right)$

f) $\cos^{-1}\left(\cos\frac{2\pi}{7}\right)$

g) $\tan^{-1}(1)$

h) $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

(d) $y = 3\sin 2(x - \frac{\pi}{3})$

$T = \frac{2\pi}{2} = \pi$

$A = 3$

phase shift $= \frac{\pi}{3}$

- $[0,\pi] \to [\frac{\pi}{3}, \pi + \frac{\pi}{3}]$
- Take $[\frac{\pi}{3}, \frac{4\pi}{3}]$
- Divide it into 4 equal parts (each of length $\frac{\pi}{4}$)
- Sketch a sine curve of amplitude 3

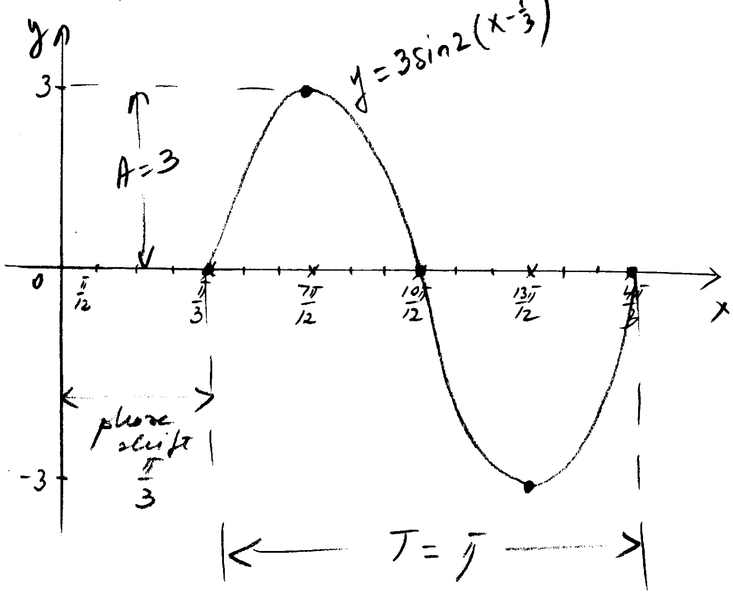
$\frac{\pi}{3} = \frac{4\pi}{12}$

$\frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$

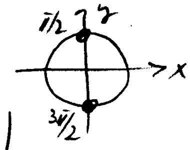
$\frac{7\pi}{12} + \frac{\pi}{4} = \frac{10\pi}{12}$

$\frac{10\pi}{12} + \frac{\pi}{4} = \frac{13\pi}{12}$

$\frac{13\pi}{12} + \frac{\pi}{4} = \frac{16\pi}{12} = \frac{4\pi}{3}$



(3) (a) $\cos x = 0$

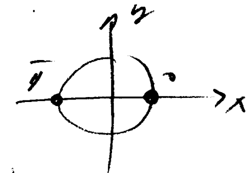


iff

$\boxed{x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}}$

(odd multiples of $\frac{\pi}{2}$)

(b) $\sin x = 0$



iff

$\boxed{x = k\pi,\ k \in \mathbb{Z}}$

(multiples of π)

(c) $\tan x = 0$ iff

$\frac{\sin x}{\cos x} = 0$ iff

$\sin x = 0$ iff

$\boxed{x = k\pi,\ k \in \mathbb{Z}}$

(d) $\cot x = 0$ iff

$\frac{\cos x}{\sin x} = 0$ iff

$\cos x = 0$ iff

$\boxed{x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}}$

(4) (a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

b/c $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3} \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

(b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

b/c $\cos\frac{3\pi}{4} = \frac{-\sqrt{2}}{2}$ and $\frac{3\pi}{4} \in [0,\pi]$

(c) $\sin^{-1}(\sin 0.4) = 0.4$

b/c $0.4 \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

and $\sin^{-1}(\sin x) = x$ for any $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

(d) $\cos\left(\sin^{-1}\frac{3}{5}\right) = ?$

let $\sin^{-1}\frac{3}{5} = u$, $u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

then $\sin u = \frac{3}{5}$

$\sin^2 u + \cos^2 u = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2 u = 1$

$\cos^2 u = 1 - \frac{9}{25}$

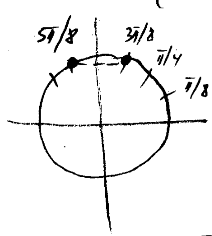
$\cos^2 u = \frac{16}{25}$

$\left.\begin{array}{l} \cos u = \pm \frac{4}{5} \\ \text{but } u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \text{ so } \cos u > 0 \end{array}\right\} \Rightarrow$

$\Rightarrow \cos u = \frac{4}{5}$

Therefore, $\boxed{\cos\left(\sin^{-1}\frac{3}{5}\right) = \frac{4}{5}}$

(e) $\sin^{-1}\left(\sin\frac{5\pi}{8}\right) \neq \frac{5\pi}{8}$ b/c



$\frac{5\pi}{8} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}\left(\sin\frac{5\pi}{8}\right) = \sin^{-1}\left(\sin\frac{3\pi}{8}\right) = \frac{3\pi}{8}$

b/c $\frac{3\pi}{8} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and $\sin^{-1}(\sin x) = x$
for any $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(f) $\cos^{-1}\left(\cos\frac{2\pi}{7}\right) = \frac{2\pi}{7}$

b/c $\frac{2\pi}{7} \in [0, \pi]$ and

$\cos^{-1}(\cos x) = x$
for any $x \in [0, \pi]$

(g) $\tan^{-1} 1 = \frac{\pi}{4}$ b/c

$\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and

$\tan\frac{\pi}{4} = 1$

(h) $\sin\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right) =$

$= \sin\left(\frac{5\pi}{6}\right)$

$= \sin\frac{\pi}{6} = \frac{1}{2}$

Math 150

Quiz #2 – SOLUTIONS

① $y = \cos x$

a) $x \in \mathbb{R}$

b) $y \in [-1, 1]$

c) $T = 2\pi$

d) $A = 1$

e) $\cos x = 0$ iff

$x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

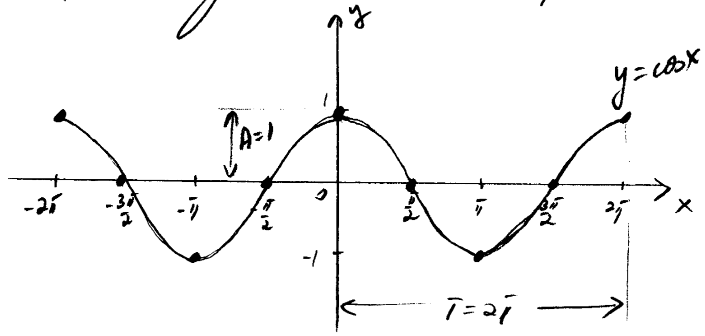
x-int: $\left(\frac{\pi}{2} + k\pi, 0\right)$

f) $x = 0,\ y = 1$

y-int: $(0, 1)$

g) even function

Graph is symmetric about y-axis

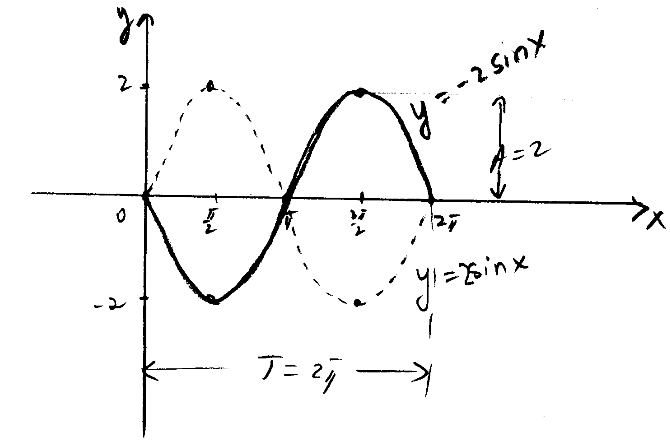


(b) $y = -2\sin x$

$T = 2\pi$

$A = 2$

- Take $[0, 2\pi]$, sketch a sine curve of amplitude 2, then reflect about x-axis

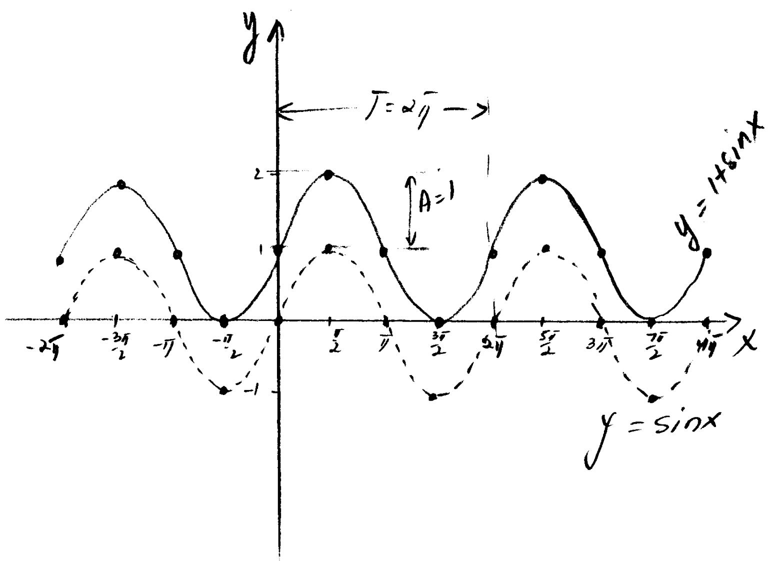


(2) (a) $y = 1 + \sin x$

$T = 2\pi$

$A = 1$

- Take $[0, 2\pi]$, graph a sine curve of amplitude 1, then shift the graph up 1 unit



(c) $y = 3\cos\frac{1}{3}x$

$T = \frac{2\pi}{\frac{1}{3}} = 6\pi$

$A = 3$

- Take $[0, 6\pi]$
- Divide it into 4 equal parts (each of length $\frac{6\pi}{4} = \frac{3\pi}{2}$)
- Sketch a cosine curve of amplitude 3