Math 71 Spring 07 Name:

TEST 3 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

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1. Solve each equation in $\mathbb{C}$ (the set of complex numbers) by the indicated method.

   a) $5(x-2)^2+38=0$ by the square root property.

   b) $3x^2-2x+5=0$ by completing the square.

   c) $x^2+\frac{x}{3}=\frac{3}{2}$ by the quadratic formula.

   d) Solve $3x^2+xy+y^2=2$ for *y*.

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2. Solve the following equations.

   a) $2x^4-x^2-3=0$

   b) $\log_4(2x-1)=3$

   c) $3^x=8$

   d) $\log_8(x+5)-\log_8 2=1$

   e) $5^x=3^{2x-1}$

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3. Solve the following inequalities.

   a) $x^2-6x+5\le 0$

   b) $\frac{1}{x+3}<\frac{1}{x-2}$

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4. Let $f(x)=3x-1$ and $g(x)=\frac{3-x}{x+1}$. Answer the following questions:

   a) Find $(g\circ f)(x)$.

   b) $(f\circ g)(2)$

   c) Find $f^{-1}(x)$.

   d) Find $g^{-1}(x)$.

5. Simplify the following expressions.

a) $4\ln x + 7\ln y - 3\ln z$

b) $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1)$

c) $\log_3 405 - \log_3 5 + \log 5 + \log 2$

d) $\log_4(\log_2 16)$

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6. For the equation given bebw, answer all the questions and graph the function (Be sure to label the axes and all points used). SHOW ALL WORK!

$$y = -2x^2 + x + 6$$

a) What type of curve is this?

b) What is the y-intercept?

c) What is the vertex

d) What are the x- intercept(s) (if any)?

e) What is the domain of the function?

f) What is the range of the function?

g) <u>Using the graph above</u>, solve the following inequality: $-2x^2 + x + 3 < 0$

h) What is the vertex form of the equation?

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7. Graph $f(x) = 2^x$ and $f^{-1}(x) = \log_2 x$ on the same coordinate system showing the symmetry about the bisector line $y = x$. Label the axes and all the points.

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8) The number of bacteria present in a culture after t hours is given by the formula $N = 1000e^{0.69t}$.

a) How many bacteria will be there after ½ hour?
b) How long will it be before there are 1,000,000 bacteria?
c) What is the doubling time

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9) The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing x baskets is $C = 0.01x^2 - 2x + 120$. How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?

(2) (a) $2x^4 - x^2 - 3 = 0$

let $x^2 = t$

then $x^4 = t^2$

then $2t^2 - t - 3 = 0$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$

$$t = \frac{1 \pm \sqrt{1+24}}{4} = \frac{1 \pm 5}{4}$$

$t = -1$ OR $t = \frac{6}{4} = \frac{3}{2}$

I if $t = -1$

$x^2 = -1$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

II if $t = \frac{3}{2}$

$x^2 = \frac{3}{2}$

$\sqrt{x^2} = \sqrt{\frac{3}{2}}$

$x = \pm \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

$x = \pm \frac{\sqrt{6}}{2}$

$\boxed{x \in \{\pm i, \pm \frac{\sqrt{6}}{2}\}}$

(b) $\log_4 (2x-1) = 3$

condition: $2x - 1 > 0$

$2x > 1$

$x > \frac{1}{2}$

$4^3 = 2x - 1$

$2x = 65$

$x = \frac{65}{2} > \frac{1}{2}$

$\boxed{x = \frac{65}{2}}$

(c) $3^x = 8$ $\quad / \log_3$

$\log_3 3^x = \log_3 8$

$\boxed{x = \log_3 8}$

OR

$3^x = 8$ $\quad / \ln$

$\ln 3^x = \ln 8$

$x \ln 3 = \ln 8$

$\boxed{x = \frac{\ln 8}{\ln 3}}$ ≈ 1.89

(d) $\log_8 (x+5) - \log_8 2 = 1$

Condition: $x + 5 > 0$

$x > -5$

$\log_8 \frac{x+5}{2} = 1$

$8^1 = \frac{x+5}{2}$

$x + 5 = 16$

$x = 9 > -5$

$\boxed{x = 9}$

(e) $5^{x} = 3^{2x-1}$ $\Big/ \ln$

$\ln 5^{x} = \ln 3^{2x-1}$

$x \ln 5 = (2x-1)\ln 3$

$x \ln 5 = 2x \ln 3 - \ln 3$

$\ln 3 = 2x \ln 3 - x \ln 5$

$\ln 3 = x(2\ln 3 - \ln 5)$

$\boxed{x = \dfrac{\ln 3}{2\ln 3 - \ln 5}} = \dfrac{\ln 3}{\ln 9 - \ln 5}$

$x \approx 1.87 \qquad = \dfrac{\ln 3}{\ln \frac{9}{5}}$

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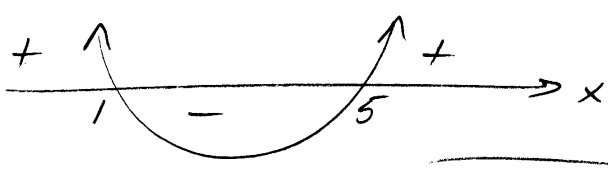
(3) (a) $x^2 - 6x + 5 \le 0$

let $y = x^2 - 6x + 5$
parabola opens up

x-in: $x^2 - 6x + 5 = 0$
$(x-1)(x-5) = 0$
$x = 1, \ x = 5$



so, $y \le 0$ iff $\boxed{x \in [1,5]}$

(b) $\dfrac{1}{x+3} < \dfrac{1}{x-2}$

$0 < \dfrac{1}{x-2}\Big/_{x+3} - \dfrac{1}{x+3}\Big/_{x-2}$

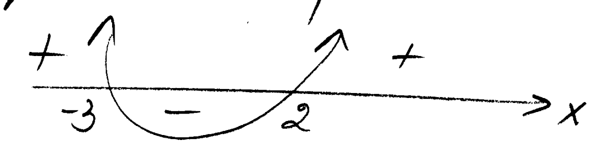
$\dfrac{x+3-(x-2)}{(x-2)(x+3)} > 0$

$\dfrac{\cancel{x}+3-\cancel{x}+2}{(x-2)(x+3)} > 0$

$\dfrac{5}{(x-2)(x+3)} > 0$ iff

$(x-2)(x+3) > 0$

let $y = (x-2)(x+3)$
parabola opens up



x-in: $x = 2, \ x = -3$

$y > 0$ iff

$\boxed{x \in (-\infty, -3) \cup (2, \infty)}$

OR table of values
(study the sign of each factor)

x	$-\infty$		-3		2		∞
$x-2$	−	−	−	−	0	+	+
$x+3$	−	−	0	+	+		+
$(x-2)(x+3)$	+			−			+

(4) $f(x) = 3x-1$

$g(x) = \frac{3-x}{x+1}$

(a) $(g\circ f)(x) = g(f(x))$

$= g(3x-1)$

$= \frac{3-(3x-1)}{(3x-1)+1} = \frac{3-3x+1}{3x-1+1}$

$\boxed{(g\circ f)(x) = \frac{4-3x}{3x}}$

(b) $(f\circ g)(2) = f(g(2))$

$g(2) = \frac{3-2}{2+1} = \frac{1}{3}$

so, $(f\circ g)(2) = f(g(2))$

$= f(\frac{1}{3})$

$= 3(\frac{1}{3}) - 1 = 0$

$\boxed{(f\circ g)(2) = 0}$

(c) $f(x) = 3x-1$

1st $y = 3x-1$

2nd $3x = y+1$ (solve for x)

$x = \frac{y+1}{3}$

3rd $y = \frac{x+1}{3}$ $(x \leftrightarrow y)$

$\boxed{f^{-1}(x) = \frac{x+1}{3}}$

(d) $g(x) = \frac{3-x}{x+1}$

1st $y = \frac{3-x}{x+1}$

2nd solve the eq. for x

$y(x+1) = 3-x$

$yx + y = 3-x$

$yx + x = 3-y$

$x(y+1) = 3-y \Rightarrow x = \frac{3-y}{y+1}$

3rd $x \leftrightarrow y$

$y = \frac{3-x}{x+1}$

$\boxed{g^{-1}(x) = \frac{3-x}{x+1}}$

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(5) (a) $4\ln x + 7\ln y - 3\ln z =$

$= \ln x^4 + \ln y^7 - \ln z^3$

$= \ln(x^4y^7) - \ln z^3$

$= \boxed{\ln \frac{x^4y^7}{z^3}}$

(b) $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1) =$

$= \frac{1}{2}\log_5 xy - \log_5(x+1)^2$

$= \log_5(xy)^{\frac{1}{2}} - \log_5(x+1)^2$

$= \boxed{\log_5 \frac{\sqrt{xy}}{(x+1)^2}}$

(c) $\log_3 405 - \log_3 5 + \log 5 + \log 2 =$

$= \log_3 \frac{405}{5} + \log(5 \cdot 2)$

$= \log_3 81 + \log 10 = 4+1 = \boxed{5}$

(d) $\log_4(\log_2 16) = \log_4 4 = \boxed{1}$

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(6) $y = -2x^2 + x + 6$

(a) parabola that opens downward $(a = -2 < 0)$

(b) let $x = 0 \Rightarrow y = 6$

$\boxed{y\text{-n}: (0,6)}$

(c) $V(x_v, y_v)$

$x_v = \frac{-b}{2a} = \frac{-1}{2(-2)} = \frac{1}{4}$

$y_v = -2 \cdot \left(\frac{1}{4}\right)^2 + \frac{1}{4} + 6$

$y_v = \frac{-1}{8} + \frac{1}{4} + 6 = \frac{1}{8} + 6 = 6\frac{1}{8}$

$\boxed{V\left(\frac{1}{4}, \frac{49}{8}\right)}$

(d) let $y = 0$

$-2x^2 + x + 6 = 0 \quad | (-1)$

$2x^2 - x - 6 = 0$

$2x^2 - x - 6 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)}$

$x = \frac{1 \pm \sqrt{1 + 48}}{4} = \frac{1 \pm 7}{4}$

$x_1 = 2, \quad x_2 = \frac{-6}{4} = \frac{-3}{2}$

$\boxed{x\text{-n}: (2,0) \text{ and } \left(\frac{-3}{2}, 0\right)}$

(e) Domain: $\boxed{x \in \mathbb{R}}$

(f) Range: $y \le \frac{49}{8}$

$\boxed{y \in \left(-\infty, \frac{49}{8}\right]}$

(g) $-2x^2 + x + 6 < 0$

iff $\boxed{x \in \left(-\infty, \frac{-3}{2}\right) \cup (2, \infty)}$

(h) $y = a(x - x_v)^2 + y_v$

$V\left(\frac{1}{4}, \frac{49}{8}\right), \quad a = -2$

$\boxed{y = -2\left(x - \frac{1}{4}\right)^2 + \frac{49}{8}}$

See graph on graphing paper

(page 7)

(7) $f(x) = 2^x$

Domain: $x \in \mathbb{R}$

x	$-\infty$	-2	-1	0	1	2	∞
$y=2^x$	$y \to 0$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	∞

H.A. $y=0$

See graph on graphing paper (page 7)

$f^{-1}(x) = \log_2 x$

Domain: $x > 0$

x	0	$\frac{1}{2}$	1	2	4	∞
$y=\log_2 x$	$-\infty$	-1	0	1	2	∞

V.A. $x=0$

(8) $N = 1000e^{0.69t}$

(a) $t = 0.5$, $N = ?$

$N = 1000e^{0.69(0.5)}$

$N \approx 1412$ bacteria after $\frac{1}{2}$ hour.

(b) $t = ?$ when $N = 1{,}000{,}000$

$1{,}000{,}\cancel{000} = \cancel{1000}e^{0.69t}$

$e^{0.69t} = 1000$ / ln

$\ln e^{0.69t} = \ln 1000$

$0.69t = \ln 1000 \Rightarrow t = \dfrac{\ln 1000}{0.69} \approx 10$ hours

(c) N_0 = initial population $= 1000e^0 = 1000$ bacteria

$t = ?$ when $N = 2N_0 = 2000$ bacteria

$2000 = 1000e^{0.69t}$

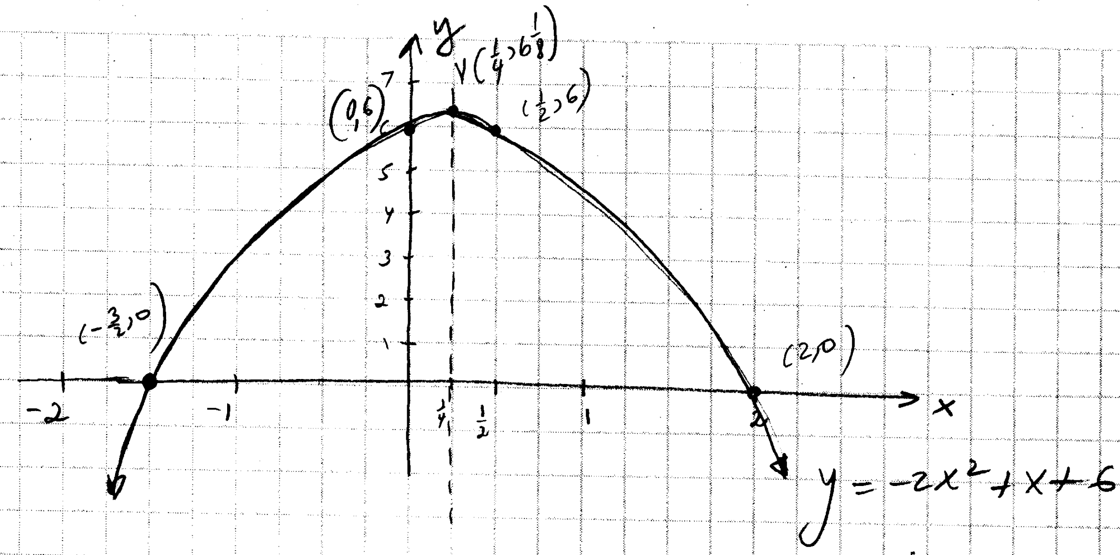
$2 = e^{0.69t}$ / ln

$\ln 2 = \ln e^{0.69t}$

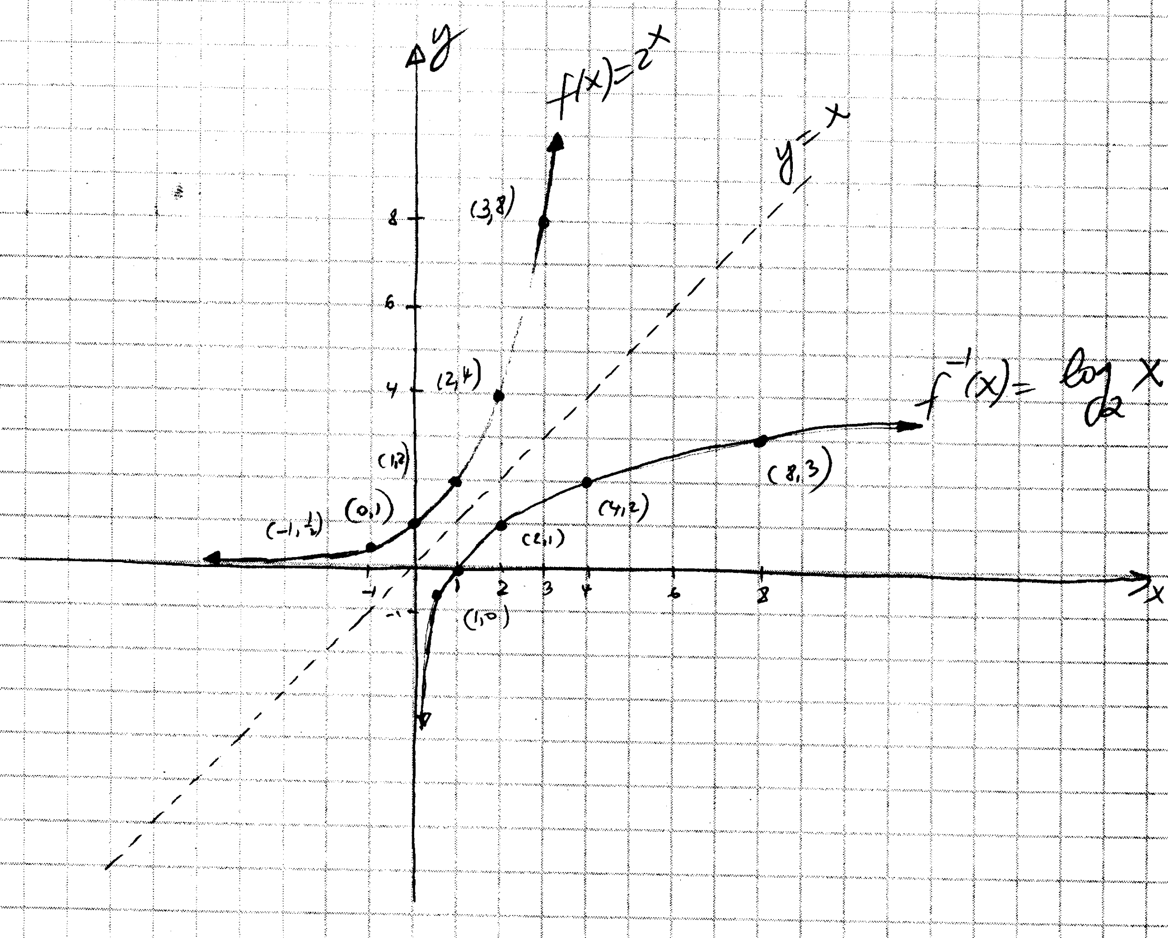
$0.69t = \ln 2 \Rightarrow t = \dfrac{\ln 2}{0.69} \approx 1$ hour

The doubling time is 1 hour.

(6)


y
x
$V(\frac{1}{4}, 6\frac{1}{8})$
(0,6)
$(\frac{1}{2}, 6)$
$(-\frac{3}{2}, 0)$
(2,0)
$y = -2x^2 + x + 6$

(7)


y
x
$f(x) = 2^x$
$y = x$
(3,8)
(2,4)
(1,2)
(0,1)
$(-1, \frac{1}{2})$
(2,1)
(4,2)
(8,3)
(1,0)
$f^{-1}(x) = \log_2 x$

(9) $C = 0.01x^2 - 2x + 120$

x = the number of baskets produced

C = cost per basket for producing x baskets

The above equation represents a parabola that opens up ↺, therefore the minimum occurs at the vertex

$V(x_v, C_v)$

$$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(0.01)} = \frac{1}{0.01} = 100 \text{ baskets}$$

$$C_v = 0.01(100)^2 - 2(100) + 120$$
$$= 1(100) - 200 + 120$$
$$= 20 \text{ \$/basket}$$

They should produce 100 baskets in order to minimize the cost per basket

Total cost at that production level (100 baskets) will be $100 \text{ baskets} \cdot (20 \text{ \$/basket})$

$$= 2000 \text{ \$}$$