Math 61 Spring 2007 Name:

TEST #1 @ 130 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.

1) Write the inverse, converse, and contrapositive of the following statement and classify the statements as true or false. If true, state the definition, postulate, or theorem your conclusion is based on. If false, say why or draw a counterexample.

Circle one Justify your choice

" *If two lines intersect and form equal adjacent angles, then they are perpendicular.* "

$P \to Q$

[✓ True] False

definition of ⊥ lines

Circle one Justify your choice

$\sim P \to \sim Q$

Inverse: if two coplanar lines don't intersect OR don't form equal adjacent angles, then they are not perpendicular

[✓ True] False

Circle one Justify your choice

$Q \to P$

Converse: if two coplanar lines are perpendicular, then they intersect and form equal adjacent angles

[✓ True] False

Circle one Justify your choice

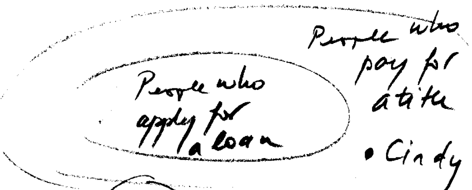
$\sim Q \to \sim P$

Contrapositive: if two coplanar lines are not perpendicular, then they don't intersect or don't form equal adjacent angles

[✓ True] False

2) Study each argument carefully to decide whether or not it is valid.

a) All people who apply for a loan must pay for a title search. $P \to Q$
Cindy paid for a title search. Q
Therefore, Cindy applied for a loan. P



VALID: YES [✓ NO]

b) If you are using this book, then you must be able to read. $P \to Q$
If you are a geometry student, you must be able to read. $R \to Q$
Therefore, if you are using this book, you are a geometry students. $P \to R$

VALID: YES [✓ NO]

3) a) Complete the following law:

$$\sim(P \wedge Q) \equiv \underline{\sim P \vee \sim Q}$$

b) Prove the law using a truth table. State clearly why we can conclude from the truth table that the law is valid.

P	Q	$\sim(P \wedge Q)$	$\sim P \vee \sim Q$
T	T	F T	F F F
T	F	T F	F T T
F	T	T F	T T F
F	F	T F	T T T

The law is valid because $\sim(P \wedge Q)$ and $(\sim P \vee \sim Q)$ have exacty the same truth values

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4) Answer true or false:

1) The hypotenuse is the side opposite one of the acute angles in a right triangle. F

2) A right isosceles triangle has two right angles. F

3) If three angles of one triangle are congruent with three angles of a second triangle, then the two triangles are congruent. F

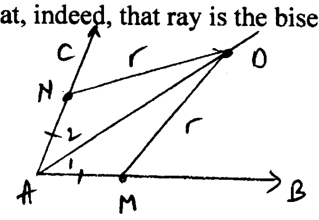
4) Triangles can be proved congruent using SSA. F

5) Corresponding parts of congruent triangles are congruent. T

6) An exterior angle of a triangle is the supplement of one of the interior angles of the triangle. T

4) **Choose only ONE of the following** . Do not do both.

Given an angle $\angle BAC$, construct using only a compass and a straightedge, the bisector $\overrightarrow{AD}$ of the given angle. Explain <u>how</u> you are constructing it and <u>then prove</u> that, indeed, that ray is the bisector of the angle.



Given: $\angle BAC$

Construct: $\overrightarrow{AD}$ - bisector

(Condition: $m\angle BAD = m\angle CAD$

Solution

1. Let $M \in \overrightarrow{AB}$

2. Mark off $N \in \overrightarrow{AC}$ such that $\overline{AN} \cong \overline{AM}$

(OR, instead of 1&2: construct circle center A and a radius r' that intersects $\overline{AB}$, $\overline{AC}$ at M and N, respectively)

3. construct circle C_1 - center M, radius r
C_2 - center N, radius r

4. $C_1 \cap C_2 = \{D\}$

5. Prove that $\overrightarrow{AD}$ = bisector of $\angle BAC$

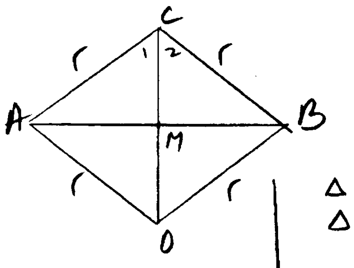
(2 points determine one line, so the construction is unique)

$\triangle ADM$, $\triangle ADN$:
$\overline{AD} \cong \overline{AD}$ reflexive $\cong$
$\overline{AM} \cong \overline{AN}$ by construction
$\overline{MD} \cong \overline{ND}$ by construction

$\triangle ADM \cong \triangle ADN$ (SSS)
$\Rightarrow \angle A_1 \cong \angle A_2$ (CPCTC)
$\Rightarrow \overrightarrow{AD}$ bisector of $\angle BAC$ (def)

OR

Given a segment, construct using only a compass and a straightedge, the midpoint of the segment. Explain <u>how</u> you are constructing it and <u>then prove</u> that, indeed, the point constructed is the midpoint of the given segment.



Given: $\overline{AB}$

Construct: M = midpoint

(Condition: $\overline{AM} \cong \overline{MB}$)
$M \in \overline{AB}$

Solution

1. Let $\overline{AB}$ - given segment

2. construct circle $\mathcal{C}_1$ - center A, $r > \frac{AB}{2}$
$\mathcal{C}_2$ - center A, r

3. $\mathcal{C}_1 \cap \mathcal{C}_2 = \{C, D\}$

4. connect C with D (2 points determine one line)

5. $\overline{CD} \cap \overline{AB} = \{M\}$

6. Prove that M = midpoint

Proof

$\triangle ACD$, $\triangle BCD$:
$\overline{CD} \cong \overline{CD}$ reflexive $\cong$
$\overline{AC} \cong \overline{BC}$ construction
$\overline{AD} \cong \overline{BD}$ (= r)

$\triangle ACD \cong \triangle BCD$ SSS
$\angle C_1 \cong \angle C_2$ CPCTC

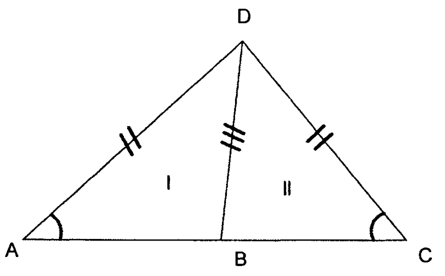
$\triangle ACM$, $\triangle BCM$:
$\overline{AC} \cong \overline{BC}$ construction
$\overline{MC} \cong \overline{MC}$ reflexive $\cong$
$\angle C_1 \cong \angle C_2$ above

$\triangle ACM \cong \triangle BCM$ SAS
$\overline{AM} \cong \overline{BM}$ CPCTC
M = midpoint of $\overline{AB}$ def. of midpt

6) i) Write the congruences given by the indicated measures or marks.

ii) State whether from the given congruences only you may conclude that triangles I and II are congruent.

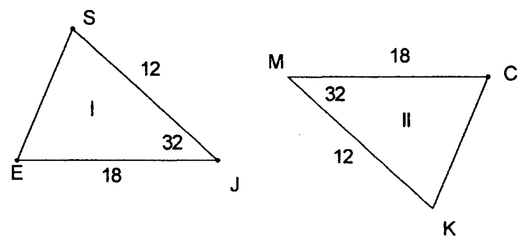
iii) If so, write what case of congruency applies.



i) $\overline{AD} \cong \overline{CD}$
$\overline{BD} \cong \overline{BD}$
$\angle A \cong \angle C$

ii) $\triangle I \ncong \triangle II$ NO

iii) N/A



i) $\overline{SJ} \cong \overline{KM}$
$\overline{EJ} \cong \overline{CM}$
$\angle J \cong \angle M$

ii) $\triangle SJE \cong \triangle KMC$ yes

iii) SAS

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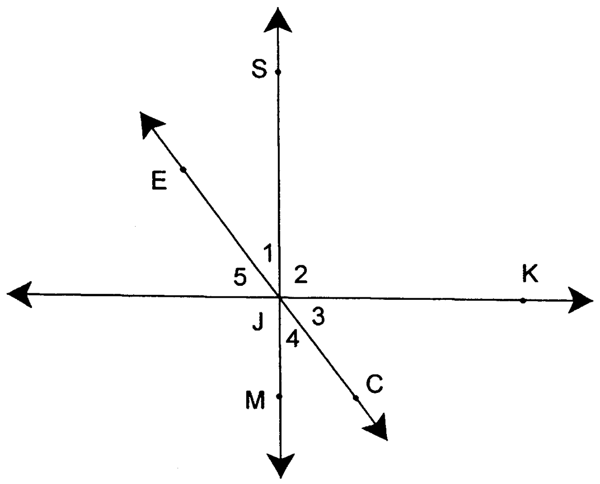
7)



Given $\overleftrightarrow{JK} \perp \overleftrightarrow{SM}$
$m\angle EJK = 105^\circ$

Find angles 1 through 5
(justify each step)

Solution

$m\angle 2 = 90^\circ$ ($\overline{JK} \perp \overline{SM}$)

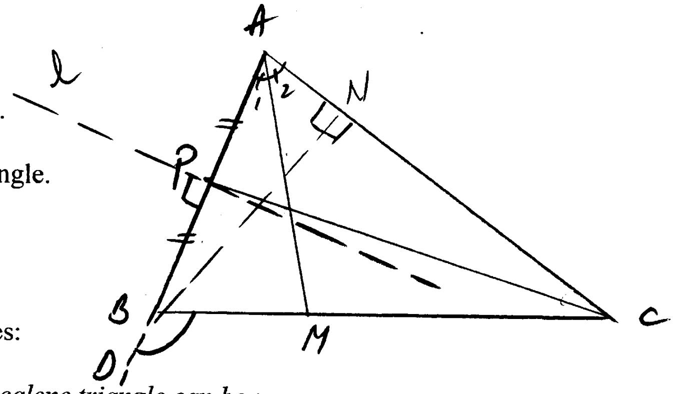
$m\angle 1 = 105^\circ - 90^\circ = 15^\circ$ (Angle Addition Postulate and Addition Prop. of =)

$m\angle 4 = m\angle 1 = 15^\circ$ (vertical ∡'s)

$m\angle 5 = 90^\circ - m\angle 1$
$= 90^\circ - 15^\circ = 75^\circ$ ($m\angle 1 + m\angle 5 = 90^\circ$)

$m\angle 3 = m\angle 5 = 75^\circ$ (vertical ∡'s)

8) A triangle ABC is given.



a) Draw a scalene triangle.

b) Check all that applies:

A scalene triangle can be :

acute ☑ right ☑ obtuse ☑ none ☐

c) Name the following:

- the angle opposite side $\overline{AC}$ $<B$
- the side opposite angle ABC $\overline{AC}$
- the angle included by $\overline{BC}$ and $\overline{AC}$ $<C$
- an exterior angle of the triangle (make sure to mark it on the drawing) $<DBC$

d) Draw the bisector of angle A, name it $\overline{AM}$, and state, using mathematical notation, that $\overline{AM}$ is the bisector of angle A (what does it mean?).

$\overline{AM}$ bisector of $\measuredangle A$ iff $m<1 = m<2$

e) Draw the altitude from vertex B to the opposite side, name it $\overline{BN}$, and state, using mathematical notation, that $\overline{BN}$ is an altitude (what does it mean?).

$\overline{BN}$ altitude iff $\overline{BN} \perp \overline{AC}$, $N \in \overleftrightarrow{AC}$

f) Draw the median from vertex C, name it $\overline{CP}$, and state, using mathematical notation, that $\overline{CP}$ is a median (what does it mean?).

$\overline{CP}$-median iff $P \in \overline{AB}$, $\overline{PA} \cong \overline{PB}$

g) Draw the perpendicular bisector of side $\overline{AB}$, name it l, and state, using mathematical notation, that l is the perpendicular bisector of $\overline{AB}$ (what does it mean?).

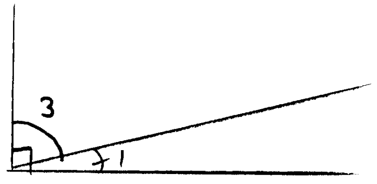
l = perpendicular bisector of $\overline{AB}$ iff $l \perp \overline{AB}$ at P where P = midpt of $\overline{AB}$

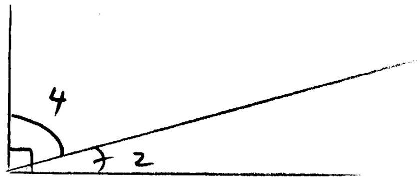
(or: $l \perp \overline{AB}$, $l \cap \overline{AB} = P$
$\overline{PA} \cong \overline{PB}$)

9) First, **complete the theorem:**

Complements of equal angles are equal in measure.

Then, **prove the theorem** (formal proof).
Make sure you state the hypothesis and conclusion of the theorem and make a drawing.





Given:
<1 and <3 are complementary
<2 and <4 are complementary
m<1 = m<2

Prove:
m<3 = m<4

Proof

1. <2 and <4 = compl. <1 and <3 = compl.	1. given
2. m<1 + m<3 = 90° m<2 + m<4 = 90°	2. def. of compl. ∠'s
3. m<1 + m<3 = m<2 + m<4	3. transitivity (substitution)
4. m<1 = m<2	4. given
5 m<1 + m<3 = m<1 + m<4	5. substitution
(3,4) 6. m<3 = m<4	6. +/− prop of =

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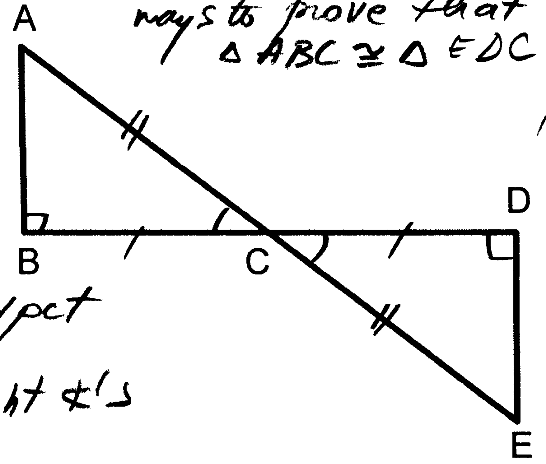
11) Given: C is the midpoint of $\overline{AE}$ and $\overline{BD}$
$\overline{AB} \perp \overline{BD}$
$\overline{DE} \perp \overline{BD}$

Prove: $\overline{AB} \cong \overline{ED}$ (formal proof)

NOTE – There are different ways to prove that △ABC ≅ △EDC { HL, HA, LA, SAS

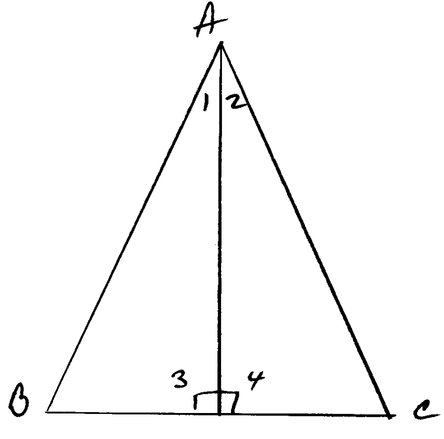


Proof

1. C = midpct $\overline{AE}$, $\overline{BD}$	1. given
2. $\overline{BC} \cong \overline{CD}$, $\overline{AC} \cong \overline{CE}$	2. def. of midpct
3. $\overline{AB} \perp \overline{BD}$, $\overline{DE} \perp \overline{BD}$	3. given
4. <B, <D = right ∠'s	4. ⊥ → right ∠'s
5. △ABC, △EDC right △'s { $\overline{AC} \cong \overline{CE}$, $\overline{BC} \cong \overline{CD}$	5. (2) above
6. △ABC ≅ △EDC	6. HL
7. $\overline{AB} \cong \overline{DE}$	7. CPCTC

12) Draw a figure and write the hypothesis and conclusion. Mark the figure and write a formal proof.

In a triangle, if an angle bisector is an altitude, then it is also a median.



Given: △ABC
$\overline{AD}$ bisector of ∡A
$\overline{AD}$ altitude

Prove: $\overline{AD}$ - median

(condition: $\overline{BD} \cong \overline{DC}$)

Proof

1. $\overline{AD}$ = altitude	1. given
2. $\overline{AD} \perp \overline{BC}$	2. def. altitude
3. $<3 \cong <4$	3 ⊥ iff ≅ adj. ∡'s
4. $\overline{AD}$ = bisector ∡A	4. given
5. $<1 \cong <2$	5. def ∡ bisector
6. △ABD, △ACD { $\overline{AD} \cong \overline{AD}$ $<1 \cong <2$ $<3 \cong <4$	6. { reflexive prop. of ≅ (5) above (3) above
7. △ABD ≅ △ACD	7. ASA (or LA)
8. $\overline{BD} \cong \overline{DC}$	8. CPCTC
9. D = midpt of $\overline{BC}$	9. def. midpt
10. $\overline{AD}$ = median	10. def median