Math 150 Spring 07

Name: Solutions

TEST #3 @ 150 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

1. a) Draw the vector $\vec{v}$ that goes from the origin to the point $(-3,5)$.
   b) Write the vector $\vec{v}$ in component form $<a,b>$.
   c) Write the vector $\vec{v}$ in terms of the unit vectors $\vec{i}$ and $\vec{j}$.
   d) Find the magnitude of the vector.

2. a) Draw the vector $\vec{v} = 5\vec{i} - 5\vec{j}$.
   b) Find the magnitude of the vector.
   c) Find the angle θ, $0^\circ \le \theta < 360^\circ$ that the vector makes with the positive x-axis.

3. Vector $\vec{v}$ is in standard position, and makes an angle of 35° with the positive x-axis. It's magnitude is 15.
   a) Write $\vec{v}$ in component form $<a,b>$.
   b) Write $\vec{v}$ in terms of the unit vectors $\vec{i}$ and $\vec{j}$.

4. a) Find the dot product of the following two vectors: $\vec{u} = 2\vec{i} - 7\vec{j}$ and $\vec{v} = -\vec{i} - 5\vec{j}$.
   b) Find the angle between the given two vectors.

5. A person is pulling a wagon with a force of 65 lbs. How much work is done in moving the wagon 150 ft if the handle makes an angle of 45° with the ground?

6. Graph the following points on a polar coordinate system:
   $A\left(1, \frac{\pi}{4}\right)$, $B\left(2, -\frac{\pi}{3}\right)$, and $C\left(-3, \frac{\pi}{6}\right)$.

7. Convert to rectangular coordinates. Use exact values.
   $\left(\sqrt{2}, -\frac{3\pi}{4}\right)$.

8. Convert to polar coordinates. Give one example with a positive r and one with a negative r.
   $(2,-2)$.

9. Write the equation with rectangular coordinates. Identify the equation (what is it?).
   $r = 4\cos\theta$

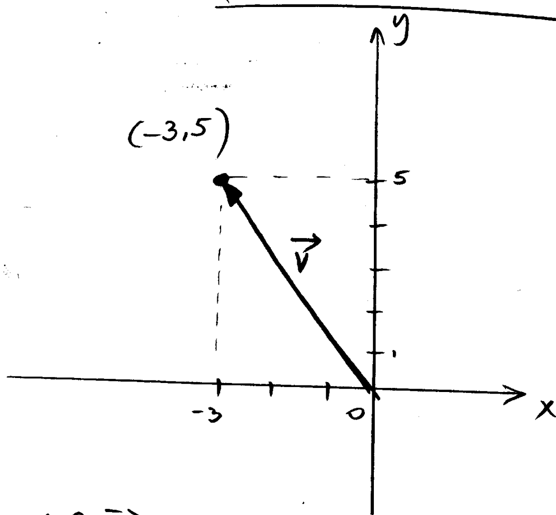
10. Sketch the graph of the given polar equation by plotting points.
    $r = 3\sin\theta$

11. Solve the triangle ABC knowing that $a = 0.48yd, b = 0.63yd, c = 0.75yd$.

Math 150

TEST # 3

① a)

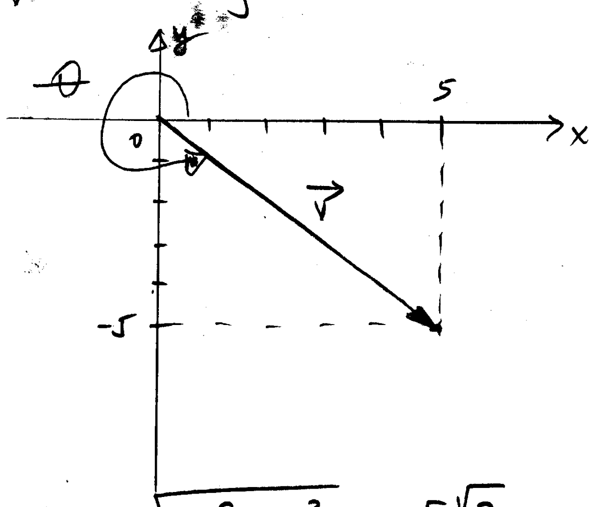


b) $\vec{v} = \langle -3, 5 \rangle$

c) $\vec{v} = -3\vec{i} + 5\vec{j}$

d) $|\vec{v}| = \sqrt{3^2+5^2} = \sqrt{34}$

② a) $\vec{v} = 5\vec{i} - 5\vec{j} = \langle 5, -5 \rangle$

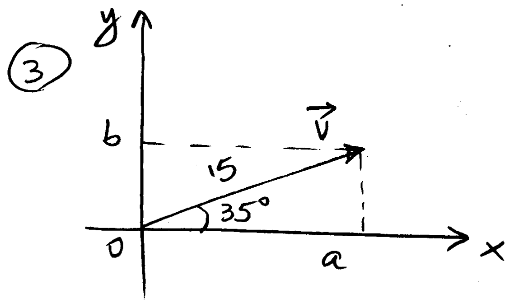


b) $|\vec{v}| = \sqrt{5^2+5^2} = 5\sqrt{2}$

c) $\tan\theta = \frac{y}{x} = -1$

$\theta = 7.45° = 315°$

③



a) $\vec{v} = \langle a, b \rangle$

$a = |\vec{v}|\cos 35° = 15\cos 35°$

$b = |\vec{v}|\sin 35° = 15\sin 35°$

$\vec{v} = \langle 15\cos 35°, 15\sin 35° \rangle$

b) $\vec{v} = 15\cos 35°\,\vec{i} + 15\sin 35°\,\vec{j}$

④ a) $\vec{u} = 2\vec{i} - 7\vec{j}$

$\vec{v} = -\vec{i} - 5\vec{j}$

$\vec{u}\cdot\vec{v} = 2(-1) + (-7)(-5) = -2+35$

$\vec{u}\cdot\vec{v} = 33$

b) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$

where θ = angle between $\vec{u}$ and $\vec{v}$, $\theta \in [0°, 180°)$

$|\vec{u}| = \sqrt{2^2+7^2} = \sqrt{53}$

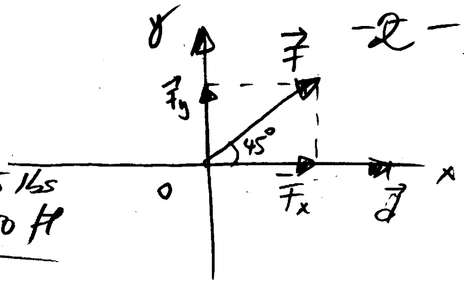
$|\vec{v}| = \sqrt{1^2+5^2} = \sqrt{26}$

$\cos\theta = \dfrac{33}{\sqrt{53}\cdot\sqrt{26}}$

$\theta = \cos^{-1}\left(\dfrac{33}{\sqrt{53}\sqrt{26}}\right)$

$\theta \approx 27.26°$

(5)



Given:
$|\vec{F}| = 65$ lbs
$|\vec{d}| = 150$ ft

Find W

$W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ = displacement vector

$\vec{d} = 150\,\vec{i}$

$\vec{F} = \vec{F}_x + \vec{F}_y$

$= |\vec{F}|\cos 45° \vec{i} + |\vec{F}|\sin 45° \vec{j}$

$= 65 \cdot \frac{\sqrt{2}}{2}\,\vec{i} + 65 \cdot \frac{\sqrt{2}}{2}\,\vec{j}$

$W = 65 \cdot \frac{\sqrt{2}}{\cancel{2}}(\overset{75}{\cancel{150}}) + 65 \cdot \frac{\sqrt{2}}{2}(0)$

$W \simeq 6894$ lbs-ft

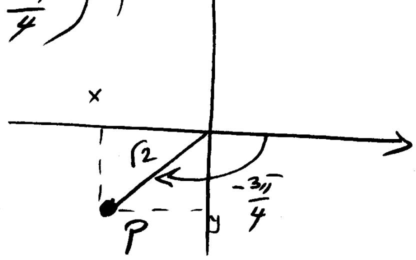
(6) see graphing paper

(7) $P(\sqrt{2}, \frac{-3\pi}{4})$ polar coordinates

$r = \sqrt{2}$

$\theta = \frac{-3\pi}{4}$

Find x, y



$x = r\cos\theta = \sqrt{2} \cdot \cos\frac{-3\pi}{4}$

$= \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -1$

$y = r\sin\theta = \sqrt{2} \cdot \sin\frac{-3\pi}{4}$

$= \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -1$

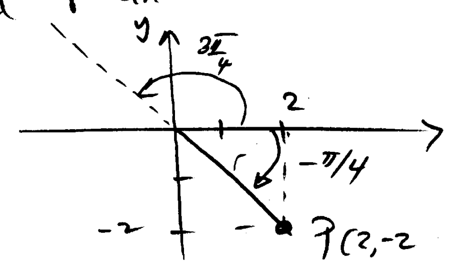
$P(-1, -1)$ rectangular coordinates

(8) $P(2, -2)$ rectangular coordinates

$x = 2$

$y = -2$

Find r and θ



$r^2 = x^2 + y^2 = 8 \Rightarrow r = \pm 2\sqrt{2}$

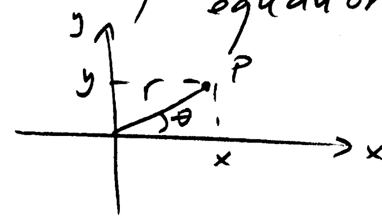
$\tan\theta = \frac{y}{x} = -1$

$\theta = \frac{-\pi}{4}$ OR $\frac{3\pi}{4}$

$P(2\sqrt{2}, -\frac{\pi}{4})$ OR $P(-2\sqrt{2}, \frac{3\pi}{4})$

(9) $r = 4\cos\theta$ polar equation

$$\begin{cases} r^2 = x^2 + y^2 \\ x = r\cos\theta \\ y = r\sin\theta \\ \tan\theta = \frac{y}{x} \end{cases}$$



$r = 4\cos\theta \quad / \cdot r$

$r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x$

$x^2 - 4x + y^2 = 0$

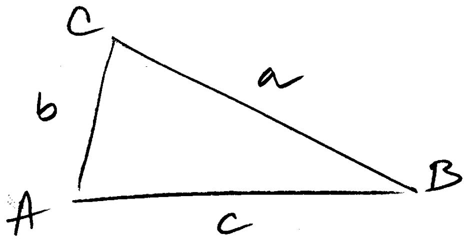
$x^2 - 4x + 4 + y^2 = 4$

$(x-2)^2 + y^2 = 4$

Circle with center $(2, 0)$

radius $\sqrt{4} = 2$

(11)



$a = 0.48$ yd
$b = 0.63$ yd
$c = 0.75$ yd

$A, B, C = ?$

Solution

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(0.63)^2 + (0.75)^2 - (0.48)^2}{2(0.63)(0.75)}$$

$$\cos A \approx 0.77 \Rightarrow A = \cos^{-1} 0.77$$

$$\boxed{A \approx 39.65^\circ}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{(0.48)^2 + (0.75)^2 - (0.63)^2}{2(0.48)(0.75)}$$

$$\cos B = 0.55 \Rightarrow B = \cos^{-1} 0.55$$

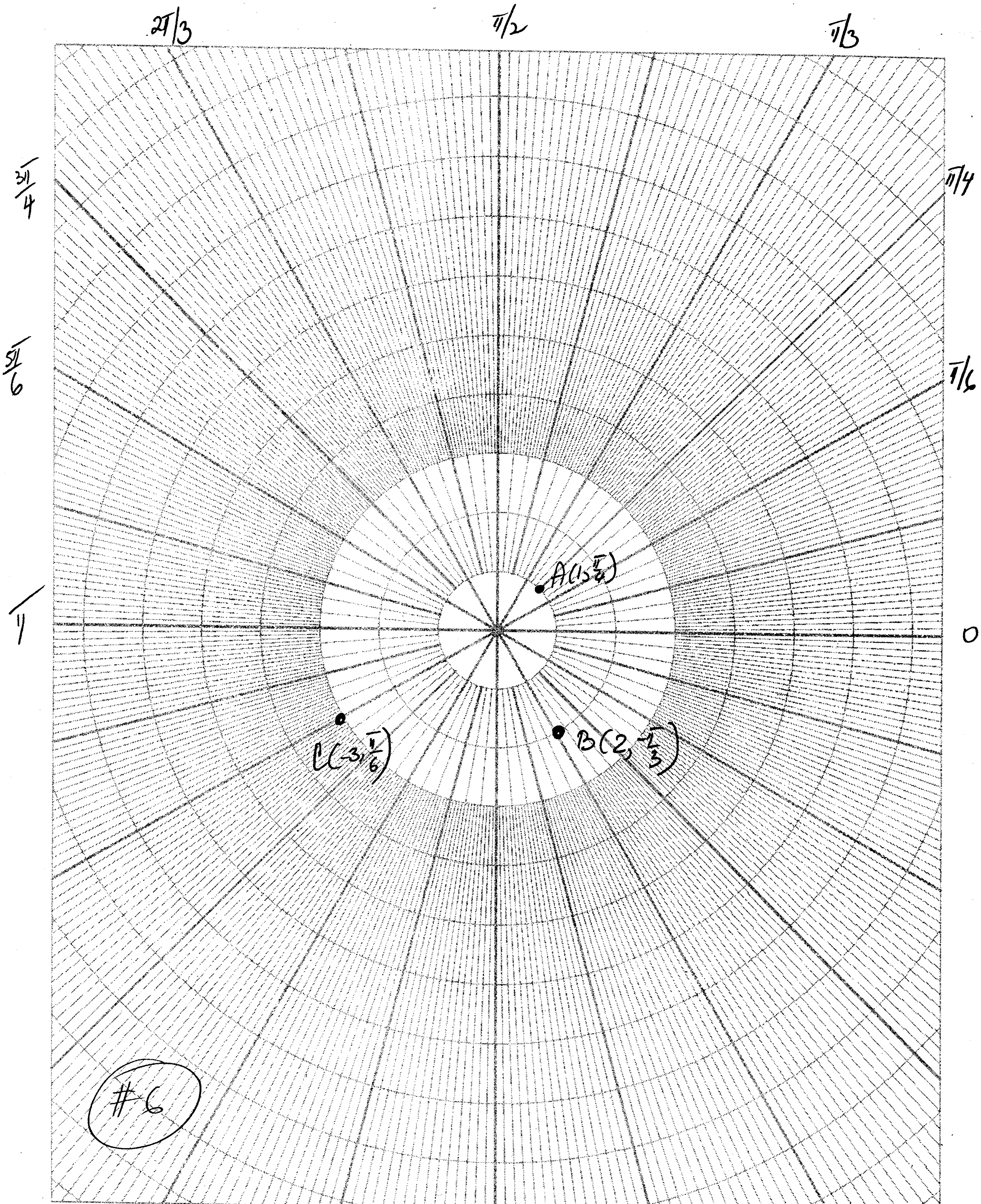
$$\boxed{B \approx 56.63^\circ}$$

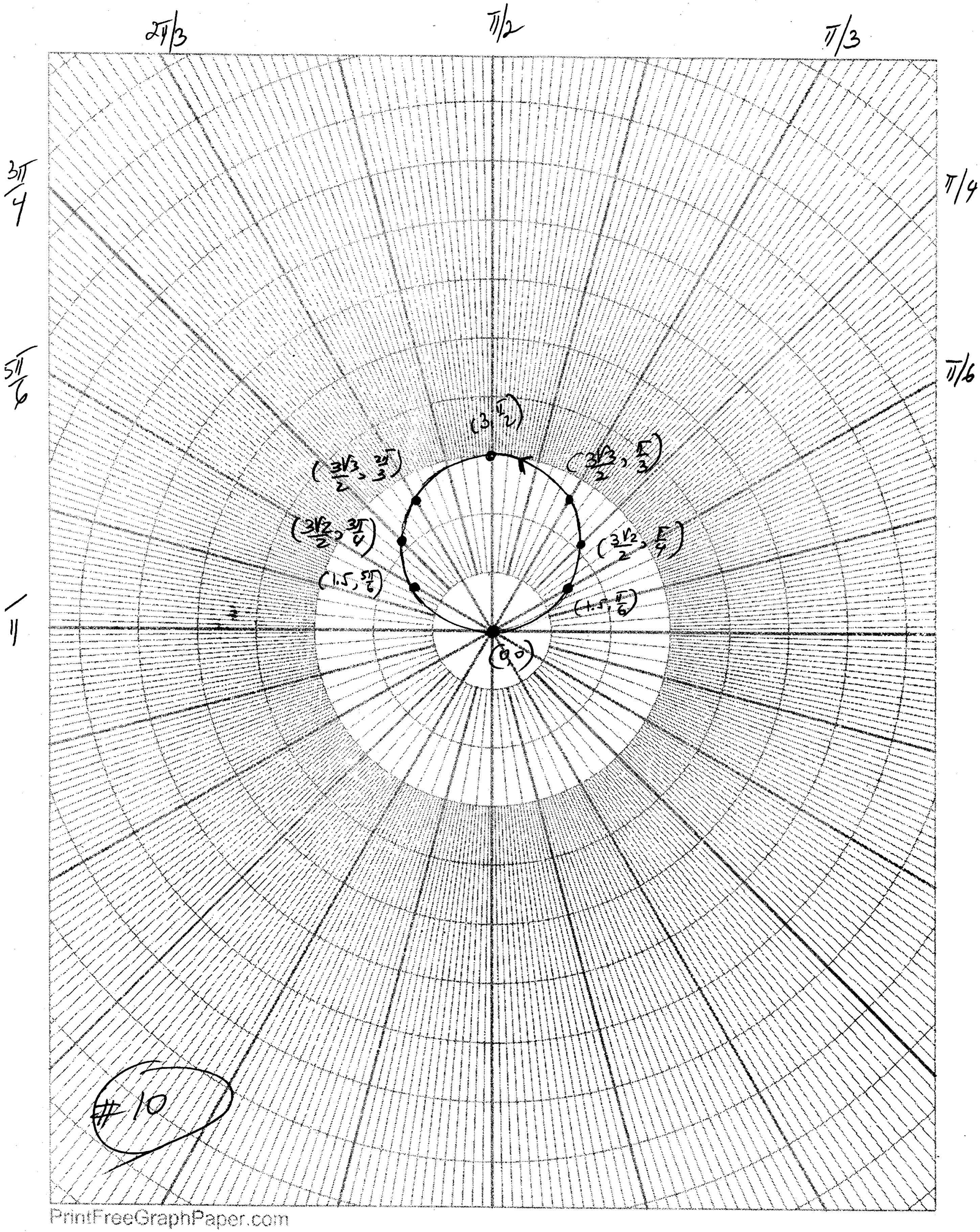
$$C = 180^\circ - A - B$$

$$\boxed{C \approx 83.72^\circ}$$

(10) $r = 3\sin\theta$

θ	$r = 3\sin\theta$	(r, θ)
0	$3\sin 0 = 0$	
$\frac{\pi}{6}$	$3\sin\frac{\pi}{6} = 3\cdot\frac{1}{2} = 1.5$	$(1.5, \frac{\pi}{6})$
$\frac{\pi}{4}$	$3\sin\frac{\pi}{4} = 3\cdot\frac{\sqrt{2}}{2} \approx 2.1$	$(\frac{3\sqrt{2}}{2}, \frac{\pi}{4})$
$\frac{\pi}{3}$	$3\sin\frac{\pi}{3} = 3\cdot\frac{\sqrt{3}}{2} \approx 2.6$	$(\frac{3\sqrt{3}}{2}, \frac{\pi}{3})$
$\frac{\pi}{2}$	$3\sin\frac{\pi}{2} = 3$	$(3, \frac{\pi}{2})$
$\frac{2\pi}{3}$	$3\sin\frac{2\pi}{3} = 3\cdot\frac{\sqrt{3}}{2} \approx 2.6$	$(\frac{3\sqrt{3}}{2}, \frac{2\pi}{3})$
$\frac{3\pi}{4}$	$3\sin\frac{3\pi}{4} = 3\cdot\frac{\sqrt{2}}{2} \approx 2.1$	$(\frac{3\sqrt{2}}{2}, \frac{3\pi}{4})$
$\frac{5\pi}{6}$	$3\sin\frac{5\pi}{6} = 3\cdot\frac{1}{2} = 1.5$	$(1.5, \frac{5\pi}{6})$
2π	$3\sin 2\pi = 0$	$(0, 2\pi)$


2π/3
π/2
π/3
3π/4
π/4
5π/6
π/6
π
0
A(1, π/4)
B(2, -π/3)
C(-3, π/6)
#6



2π/3
π/2
π/3
3π/4
π/4
5π/6
π/6
π
(3, π/2)
(3√3/2, 2π/3)
(3√3/2, π/3)
(3√2/2, 3π/4)
(3√2/2, π/4)
(1.5, 5π/6)
(1.5, π/6)
(0,0)
#10