Math 71 Spring 2006 QUIZ #3 @ 15 points Name: SOLUTIONS

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given.

1. Find an equation of the line satisfying each of the conditions:

a) slope 5 and passing through $(1,-4)$;

$\begin{cases} m=5 \\ (1,-4) \end{cases}$

$y-y_1 = m(x-x_1)$

$y-(-4) = 5(x-1)$

$\boxed{y+4 = 5(x-1)}$

OR

$y+4 = 5x-5$

$\boxed{y = 5x-9}$

b) passing through $(1,-3)$ and $(2,-4)$.

We need $\begin{cases} \text{slope } m = ? \\ \text{a point } (1,-3) \end{cases}$

$y-y_1 = m(x-x_1)$

$m = \frac{\Delta y}{\Delta x} = \frac{-3-(-4)}{1-2} = \frac{-3+4}{-1} = \frac{1}{-1} = -1$

$m=-1$

$y-(-3) = -1(x-1)$

$\boxed{y+3 = -x+1}$ OR

$\boxed{y = -x-2}$

2) Are the lines given by these equations parallel, perpendicular or neither?

$$2y - \frac{1}{3}x = 0$$

$$4x + 6y = 1.$$

We know that $l_1 \parallel l_2 \iff m_1 = m_2$

$l_1 \perp l_2 \iff m_1 m_2 = -1$

Let's find the slope of each line:

(l_1) $2y - \frac{1}{3}x = 0$

$2y = \frac{1}{3}x \quad / \cdot \frac{1}{2}$

$y = \frac{1}{2}\cdot\frac{1}{3}x$

$y = \frac{1}{6}x$

$m_1 = \frac{1}{6}$

(l_2) $4x + 6y = 1$

$6y = -4x+1 \quad / : 6$

$y = \frac{-4}{6}x + \frac{1}{6}$

$y = \frac{-2}{3}x + \frac{1}{6}$

$m_2 = \frac{-2}{3}$

$m_1 \neq m_2 \Rightarrow$ the lines are not parallel

$m_1 m_2 \neq -1 \Rightarrow$ the lines are not perpendicular

3. Let $f(x) = 3x+1$ and $g(x) = \frac{x}{2}$ two functions.

a) Find $(f \circ g)(x) = f(g(x))$

$= f(\frac{x}{2})$

$= 3 \cdot \frac{x}{2} + 1$

$= \frac{3}{2}x + 1$

$\boxed{(f \circ g)(x) = \frac{3}{2}x + 1}$

b) $(g \circ f)(-1) = g(f(-1))$

$= g(-3+1)$

$= g(-2)$

$= \frac{-2}{2}$

$= -1$

$\boxed{(g \circ f)(-1) = -1}$

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4. Given $f(x) = \frac{2x-5}{3}$, find $f^{-1}(x)$.

1. Let $y = \frac{2x-5}{3}$

2. Solve the equation for x:

$3y = 2x - 5$

$2x = 3y + 5$

$x = \frac{3y+5}{2}$

3. $x \leftrightarrow y$

$y = \frac{3x+5}{2}$

$\boxed{f^{-1}(x) = \frac{3x+5}{2}}$

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5. Determine whether the given functions are inverses of each other:

$f(x) = \frac{3}{x-4}$ and $g(x) = \frac{3}{x} + 4$.

Two functions f and g are inverses of each other if and only if ① $(f \circ g)(x) = x$ and

② $(g \circ f)(x) = x$

① $(f \circ g)(x) = f(g(x))$

$= f(\frac{3}{x} + 4)$

$= \frac{3}{\frac{3}{x} + 4 - 4}$

$= \frac{3}{\frac{3}{x}} = 3 \div \frac{3}{x}$

$= 3 \cdot \frac{x}{3} = x$

$(f \circ g)(x) = x$

② $(g \circ f)(x) = g(f(x))$

$= g(\frac{3}{x-4})$

$= \frac{3}{\frac{3}{x-4}} + 4$

$= 3 \div \frac{3}{x-4} + 4$

$= \cancel{3} \cdot \frac{x-4}{\cancel{3}} + 4$

$= x - 4 + 4$

$= x$

$(g \circ f)(x) = x$

Therefore f and g are inverses of each other