

Write in a neat and organized fashion. Use a pencil. Show all work to get credit.

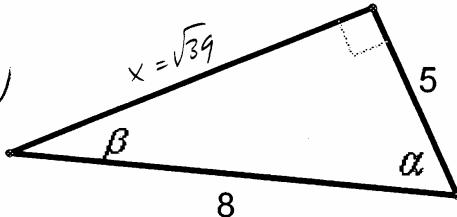
SOLUTIONS

1. Find  $\sin \alpha$ ,  $\cos \beta$ ,  $\tan \alpha$ ,  $\cot \beta$  if

Let  $x = \text{unknown leg}$   
Then  $x^2 = 8^2 - 5^2$  (Pythagorean theorem)

$$x^2 = 39$$

$$x = \sqrt{39}$$



$$\sin \alpha = \frac{\sqrt{39}}{8}$$

$$\tan \alpha = \frac{\sqrt{39}}{5}$$

$$\cos \beta = \frac{\sqrt{39}}{8}$$

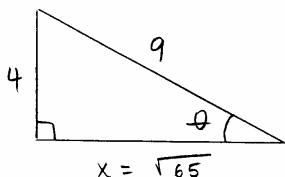
$$\cot \beta = \frac{\sqrt{39}}{5}$$

2. Sketch a right triangle that has one acute angle  $\theta$ , and find the other five trigonometric ratios of  $\theta$  knowing that

$$\sin \theta = \frac{4}{9}$$

Let  $x = \text{unknown leg}$   
 $x^2 = 9^2 - 4^2$  (Pythagorean theorem)

$$x^2 = 65 \Rightarrow x = \sqrt{65}$$



$$\cos \theta = \frac{\sqrt{65}}{9}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{9}{\sqrt{65}} = \frac{9\sqrt{65}}{65}$$

$$\tan \theta = \frac{4}{\sqrt{65}} = \frac{4\sqrt{65}}{65}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{9}{4}$$

$$\cot \theta = \frac{\sqrt{65}}{4}$$

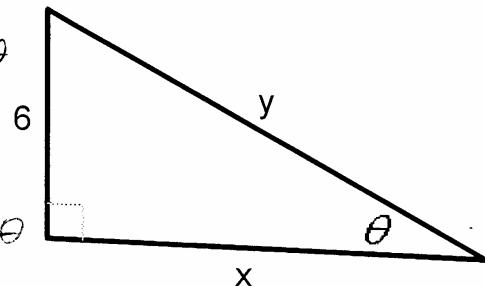
3. Express  $x$  and  $y$  in terms of trigonometric ratios of  $\theta$ .

x  $\tan \theta = \frac{6}{x} \Rightarrow x = \frac{6}{\tan \theta}$  OR  $x = 6 \cot \theta$

OR

$$\cot \theta = \frac{x}{6} \Rightarrow x = 6 \cot \theta$$

y  $\sin \theta = \frac{6}{y} \Rightarrow y = \frac{6}{\sin \theta}$  OR  $y = 6 \csc \theta$



4. Simplify the following expressions:

$$\begin{aligned}
 \text{a) } & \cos u + \tan u \sin u = \\
 & = \cos u + \frac{\sin u}{\cos u} \cdot \frac{\sin u}{1} \\
 & = \cos u + \frac{\sin^2 u}{\cos u} \\
 & = \frac{\cos^2 u + \sin^2 u}{\cos u} = \boxed{\frac{1}{\cos u} = \sec u}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \tan x \cos x \csc x = \\
 & = \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} \cdot \frac{1}{\sin x} \\
 & = 1
 \end{aligned}$$

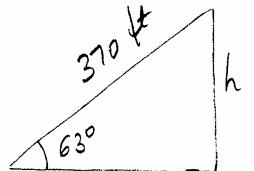
5. Verify the identity.

$$\begin{aligned}
 \cos \theta (\sec \theta - \cos \theta) &= \sin^2 \theta \\
 \cos \theta (\sec \theta - \cos \theta) &= \cos \theta \left( \frac{1}{\cos \theta} - \cos \theta \right) \\
 &= 1 - \cos^2 \theta \quad \left( \text{b/c } \sin^2 \theta + \cos^2 \theta = 1 \right. \\
 &\quad \left. \sin^2 \theta = 1 - \cos^2 \theta \right) \\
 &= \sin^2 \theta
 \end{aligned}$$

6. Show that the equation is not an identity.

$$\begin{aligned}
 \sin x - \cos x &= 0 \\
 \text{if } x = 30^\circ, \text{ then } \sin 30^\circ - \cos 30^\circ &= \\
 &= \frac{1}{2} - \frac{\sqrt{3}}{2} \neq 0 \\
 \text{Therefore, } \sin x - \cos x &= 0 \text{ is not an identity.}
 \end{aligned}$$

7. A man is lying on the beach, flying a kite. He holds the end of the kite string at ground level, and estimates the angle of elevation of the kite to be  $63^\circ$ . If the string is 370 ft long, how high is the kite above the ground?



$$\begin{aligned}
 \text{let } h &= \text{height} \\
 \sin 63^\circ &= \frac{h}{370} \\
 h &= 370 \sin 63^\circ \\
 h &\approx 330 \text{ ft}
 \end{aligned}$$