DUE 04/11/06 PROJECT #3 - 15 POINTS

The Pythagorean Theorem

Name(s): ____________________

In this investigation, you'll create a custom tool for constructing a square, then you'll construct squares on the sides of a right triangle. The areas of these squares illustrate perhaps the most famous relationship in mathematics—the Pythagorean theorem.

Sketch and Investigate

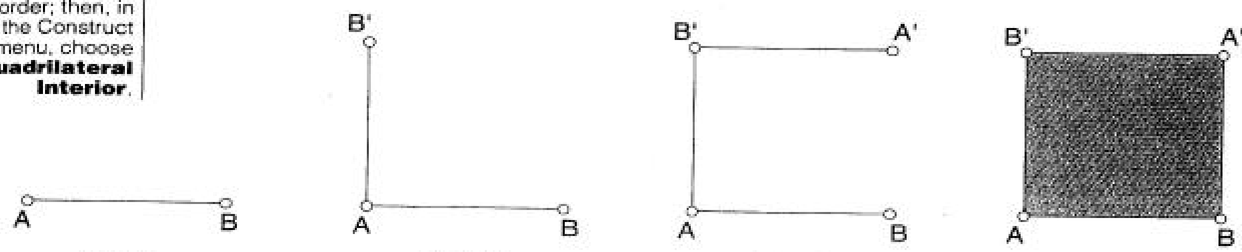
1. Construct $\overline{AB}$.

> Double-click point A to mark it as a center. Select point B and $\overline{AB}$; then, in the Transform menu, choose **Rotate**.

2. Mark point A as a center and rotate point B and $\overline{AB}$ by 90°.
3. Mark point B' as a center and rotate point A and $\overline{B'A}$ by 90°.
4. Construct $\overline{A'B}$ to finish the square.

> Select the vertices in consecutive order; then, in the Construct menu, choose **Quadrilateral Interior**.

5. Construct the interior.



Step 1 Step 2 Step 3 Steps 4 and 5

6. Drag each vertex of the square to make sure it holds together.

> Use the **Text** tool and click on each point.

7. Hide the labels.

> Select the entire figure; then, in the Custom Tools menu (the bottom tool in the Toolbox), choose **Create New Tool**.

8. Make a custom tool of this construction.

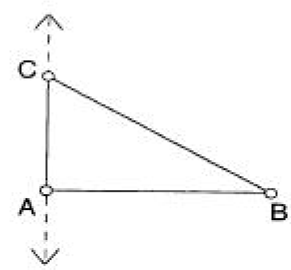
Q1 What properties of a square did you use in this construction?

> You might want to save this tool so that you can use it in the future. See the appropriate sections in the help system. (Choose **Toolbox** from the Help menu, then click on the Custom Tools link.)

9. Experiment with using the custom tool to get a feel for the way it works. Note that the direction in which the square is constructed depends on how you use the tool.
10. Open a new sketch.
11. Construct $\overline{AB}$.

> Select point A and $\overline{AB}$ and, in the Construct menu, choose **Perpendicular Line**.

12. Construct a line through point A perpendicular to $\overline{AB}$.
13. Construct $\overline{BC}$, where point C is a point on the perpendicular line.

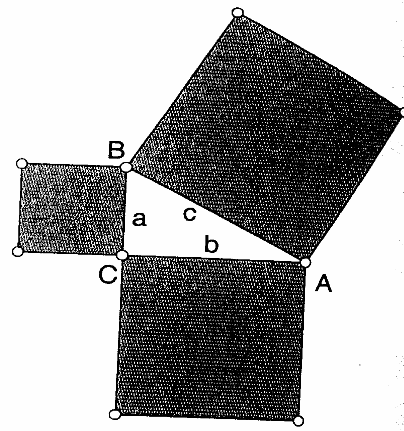


14. Hide the perpendicular line and construct $\overline{AC}$.
15. Drag each vertex to confirm that your triangle stays a right triangle.

Q2 What property of a right triangle did you use in your construction?

To change a label, double-click the label with the finger of the **Text** tool. (The reason for changing these labels is so that your figure will match the way the theorem is usually stated. This may make it easier to remember the theorem.)

16. Change labels so that the right-angle vertex is labeled C and the other two vertices are labeled A and B.
17. Show the labels of the sides. Change them to a, b, and c so that side a is opposite $\angle A$, side b is opposite $\angle B$, and side c is opposite $\angle C$.



Be sure to attach each square to a pair of the triangle's vertices. If your square goes the wrong way (overlaps the interior of your triangle) or is not attached properly, undo and try attaching the square to the triangle's vertices in the opposite order.

18. Use your square tool to construct squares on the sides of your triangle.
19. Drag the vertices of the triangle to make sure the squares are properly attached.
20. Measure the areas of the three squares.
21. Measure the lengths of sides a, b, and c.
22. Drag each vertex of the triangle and observe the measures.

Choose **Calculate** from the Measure menu to open the Calculator. Click once on a measurement to enter it into a calculation.

Q3 Describe any relationship you see among the three areas. Use the Calculator to create an expression that confirms your observations.

Q4 Based on your observations about the areas of the squares, write an equation that relates a, b, and c in any right triangle. (*Hint:* What's the area of the square with side length a? What are the areas of the squares with side lengths b and c? How are these areas related?)

Explore More

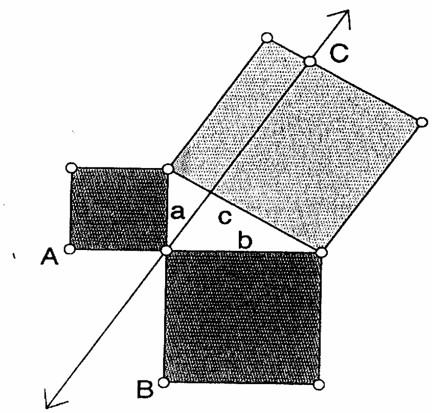
1. Do a similar investigation using other figures besides squares. Does your conjecture about the areas still hold?
2. Investigate the converse of the Pythagorean theorem: Construct a nonright triangle and squares on its sides. Measure the areas of the squares and sum two of them. Drag until the sum is equal to the third area. What kind of triangle do you have?

Visual Demonstration of the Pythagorean Theorem

Name(s): ____________________

In this activity, you'll do a visual demonstration of the Pythagorean theorem based on Euclid's proof. By *shearing* the squares on the sides of a right triangle, you'll create congruent shapes without changing the areas of your original squares.

Sketch and Investigate



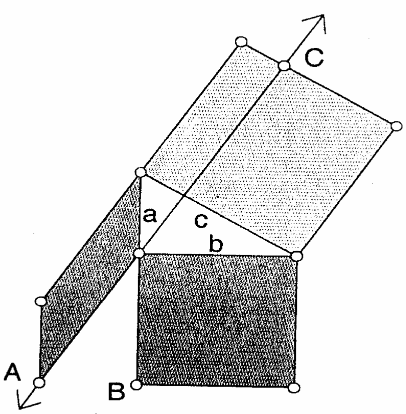
1. Open the sketch **Shear Pythagoras.gsp.** You'll see a right triangle with squares on its sides.

Click on an interior to select it. Then, in the Measure menu, choose **Area**.

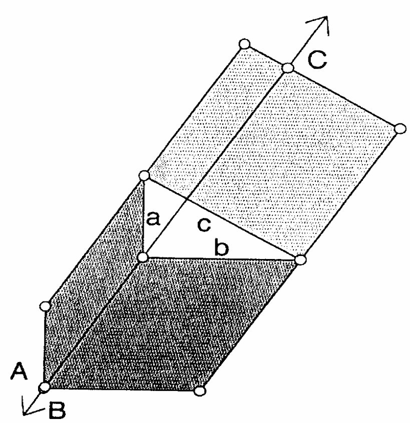
2. Measure the areas of the squares.
3. Drag point *A* onto the line that's perpendicular to the hypotenuse. Note that as the square becomes a parallelogram its area doesn't change.
4. Drag point *B* onto the line. It should overlap point *A* so that the two parallelograms form a single irregular shape.

To confirm that this shape is congruent, you can copy and paste it. Drag the pasted copy onto the shape on the legs to see that it fits perfectly.

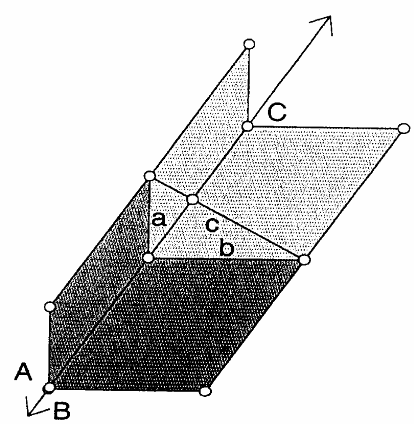
5. Drag point *C* so that the large square deforms to fill in the triangle. The area of this shape doesn't change either. It should appear congruent to the shape you made with the two smaller parallelograms.



Step 3



Step 4



Step 5

To confirm that this works for any right triangle, change the shape of the triangle and try the experiment again.

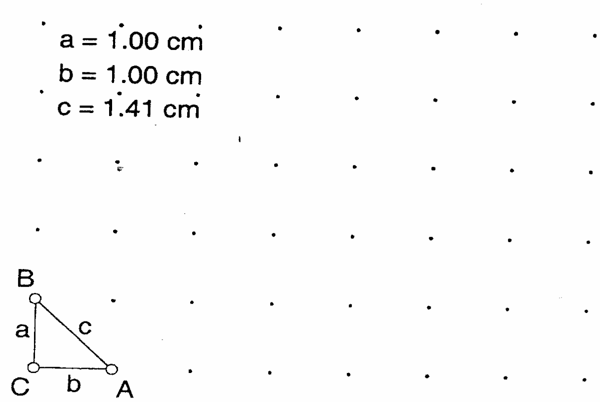
Q1 How do these congruent shapes demonstrate the Pythagorean theorem? (*Hint:* If the shapes are congruent, what do you know about their areas?)

Pythagorean Triples

Name(s): ______________________

The Pythagorean theorem states that if a right triangle has side lengths a and b and hypotenuse length c, then $a^2 + b^2 = c^2$. A set of three whole numbers that satisfy the Pythagorean theorem is called a Pythagorean triple. In this activity, you'll find as many right triangles as you can whose side lengths are whole numbers.

Sketch and Investigate



1. Make your sketch window as large as you can.

In the Edit menu, choose **Preferences** and go to the Units panel.

2. In Preferences, set the Distance Units to **cm** and the Distance Precision to **hundredths**.

In the Graph menu, hoose **Show Grid**, then choose **Snap Points**. Select the grid (by clicking on a grid intersection) and choose **Display | Line Width | Dotted**.

3. Show the grid and turn on point snapping.
4. Hide the axes and the two control points.
5. In the lower left corner of your sketch, draw a right triangle ABC with vertices on the grid.

sing the **Text** tool, click on a segment to show its label. Double-click a label to change it.

6. Show the segment labels and change them to a, b, and c, as shown.
7. Measure the three side lengths.
8. Make the two leg lengths 1 cm each, as shown.

Q1 In this case, you can see that the hypotenuse length is not a whole number. Use the Pythagorean theorem to find the exact hypotenuse length (in radical form) when the side lengths are 1 cm. Show your work.

9. Drag point A one unit to the right.

Q2 When the leg lengths are 1 cm and 2 cm, is the hypotenuse length a whole number?

10. Drag point A one unit to the right again and look to see if the hypotenuse length is a whole number.

You may or may not be able to fill in the whole chart, depending on the thoroughness of your search and the size of your screen. If your screen is very large, you may even need to add rows to the chart.

Q3 Continue a systematic search for Pythagorean triples, dragging point *A* one unit at a time to the right to increase *b* and dragging point *B* one unit up to increase *a*. Any time *c* is a whole number, record the Pythagorean triple in the chart at right.

a	*b*	*c*

Refer to your chart and experiment with the sketch to answer the following questions:

Q4 Which sets of triples are side lengths of congruent triangles?

Q5 Which sets of triples are side lengths of similar triangles (triangles with the same shape)?

Q6 Do you think there is a limit to the number of Pythagorean triples possible? Explain.

Q7 In the space below, use the Pythagorean theorem to verify at least three of your sets of triples.

Explore More

1. Euclid's *Elements* demonstrates that Pythagorean triples can be generated by the formulas $m^2 - n^2$, $2mn$, $m^2 + n^2$, where m and n are positive integers and m is greater than n. What triple is generated by $m = 2$ and $n = 1$? Increase m and n and generate some other triples. Can you generate all the triples you recorded in your chart? Can you generate some triples that aren't on your chart? Draw some triangles with these side lengths on the grid to confirm that they're right triangles.