PROJECT #2

Medians in a Triangle

Question 1: The third median intersects the other two at their point of intersection. The three medians are concurrent.

Question 2: $\frac{BCe}{CeF} = 2$ or $\frac{CeF}{BCe} = \frac{1}{2}$

Question 3: The centroid divides each median into two parts. One part is twice the length of the other.

Question 4: The slope of the line is ½ (or 2, depending on which way you built the table). This represents the ratio between the lengths of the segments the centroid creates on a median.

Perpendicular Bisectors in a Triangle

Question 1: The third perpendicular bisector intersects the other two at their point of intersection. The three perpendicular bisectors are concurrent.

Question 2: Point G is outside the triangle when the triangle is obtuse. Point G is inside the triangle when the triangle is acute.

Question 3: When point G is on the triangle, the triangle is right. Point G lies on the midpoint of the hypotenuse.

Question 4: The distances are the same from the circumcenter to each vertex.

Explore more

#2: The circumcenter is equidistant from vertices A and B because it lies on the perpendicular bisector of $\overline{AB}$. It is equidistant from vertices B and C because it lies on the perpendicular bisector of $\overline{BC}$. Therefore, using the transitive property, it is equidistant from A and B and C.

#3: Unlike triangles, not all quadrilaterals can be circumscribed. Only those whose perpendicular bisectors are concurrent.

Altitudes in a Triangle

Question 1: If an altitude falls outside the triangle, the triangle is obtuse.

Question 2: When angle A is right, altitude BD lies on side BA.

Question 3: When the triangle is acute, the three altitudes intersect at the same point inside the triangle.

Question 4: When the triangle is obtuse, two altitudes fall outside the triangle and the third one falls inside.

Question 5: All three lines containing the altitudes always intersect at the orthocenter of the triangle. The orthocenter lies inside an acute triangle and outside an obtuse triangle.

Angle Bisectors in a Triangle

Question 1: The third angle bisector intersects the other angle bisectors at their point of intersection.

Question 2: The distance from the incenter to each of the three sides of a triangle is the same.

Explore more

#3: Any point on an angle bisector is equidistant from two sides of the angle being bisected. The incenter is on all three angle bisectors, so it is equidistant from all three sides of the triangle. Thus, a circle whose radius is this distance will just touch the three sides.

#4: You can inscribe a circle in any quadrilateral with concurrent angle bisectors. This type of quadrilateral includes rhombuses, kites, squares, but not most rectangles or parallelograms.

The Euler Segment

Question 1: The orthocenter, the centroid, and the circumcenter are always collinear.

Question 2: For example:

- in an equilateral triangle, all four points are coincident;
- in an isosceles triangle, all four points are collinear and lie along the median to the vertex angle;
- in acute triangle, all four points lie inside the triangle;
- in an obtuse triangle, the circumcenter and orthocenter lie outside the triangle;
- in a right triangle, the orthocenter lies on the vertex of the right angle, and the circumcenter lies on the midpoint of the hypotenuse.

Question 3: The circumcenter and the orthocenter are the endpoints of the Euler segment. The centroid lies between them.

Question 4: The distance from the orthocenter to the centroid is twice the distance from the centroid to the circumcenter.

Explore more

#1: Three of the points are the midpoints of the sides of the original triangle. Three other points are the points where the altitudes intersect the opposite sides of the triangle. The last three points are the midpoints of the segments connecting the orthocenter with each vertex.

#2:

- In an equilateral triangle, three pairs of points coincide, reducing the nine points to six.
- In a right triangle, three points coincide at the right angle vertex, and a pair of points coincides at each of the two side midpoints, so the nine points are reduced to five.
- In an isosceles right triangle, the foot of the third altitude coincides with the midpoint of a side, so the five points are reduced to four.