Math 130 Spring 2006

Name: SOLUTIONS

# TEST 1 @ 140 points

Each exercises is worth 15.50 points (15 for proof and .50 for neatness and good mathematical format). Complete steps must be shown for each exercise. Do not just write down an answer. **No proof, no credit given!**

## PART I

Solve all exercises

1. Let $A(2,-1)$ and $B(-3,1)$ be two points in a plane.

a) Find an equation of the circle with diameter $AB$ (note that the diameter is twice the radius).

Center = midpoint of $AB$

Center $(x_M, y_M)$

$x_M = \frac{x_A + x_B}{2} = \frac{2+(-3)}{2} = \frac{-1}{2}$

$y_M = \frac{-1+1}{2} = \frac{0}{2} = 0$

$\boxed{\text{Center}\left(\frac{-1}{2}, 0\right)}$

radius $= \frac{AB}{2}$

$AB^2 = (\Delta x)^2 + (\Delta y)^2$

$= (2-(-3))^2 + (-1-1)^2$

$= 5^2 + (-2)^2$

$= 25 + 4$

$AB^2 = 29$

$AB = \sqrt{29}$

$\boxed{r = \frac{\sqrt{29}}{2}}$

Circle:

$\left(x - \left(\frac{-1}{2}\right)\right)^2 + (y-0)^2 = \left(\frac{\sqrt{29}}{2}\right)^2$

$\boxed{\left(x + \frac{1}{2}\right)^2 + y^2 = \frac{29}{4}}$

b) Find the <u>exact</u> x-intercepts (if any).

$\left(x + \frac{1}{2}\right)^2 + y^2 = \frac{29}{4}$

Let $y = 0$

$\left(x + \frac{1}{2}\right)^2 = \frac{29}{4}$

$\sqrt{\left(x + \frac{1}{2}\right)^2} = \sqrt{\frac{29}{4}}$

$x + \frac{1}{2} = \pm \frac{\sqrt{29}}{2}$

$x = \frac{-1}{2} \pm \frac{\sqrt{29}}{2}$

The x-n are $\boxed{\left(\frac{-1}{2} \pm \frac{\sqrt{29}}{2}, 0\right)}$

c) Find the <u>exact</u> y-intercepts (if any).

$\left(x + \frac{1}{2}\right)^2 + y^2 = \frac{29}{4}$

Let $x = 0$

$\left(\frac{1}{2}\right)^2 + y^2 = \frac{29}{4}$

$\frac{1}{4} + y^2 = \frac{29}{4}$

$y^2 = \frac{28}{4}$

$y^2 = 7$

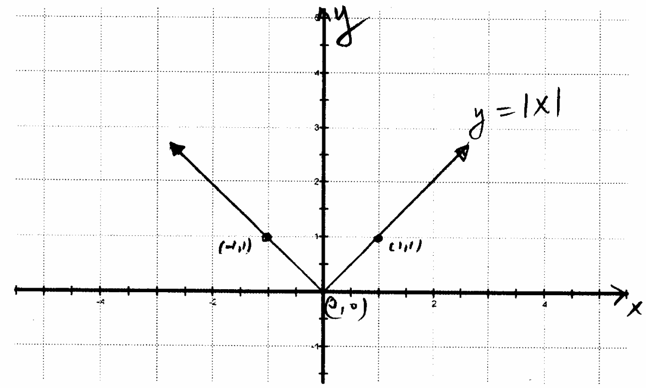
$\sqrt{y^2} = \sqrt{7}$

$y = \pm\sqrt{7}$

The y-n are $\boxed{(0, \pm\sqrt{7})}$

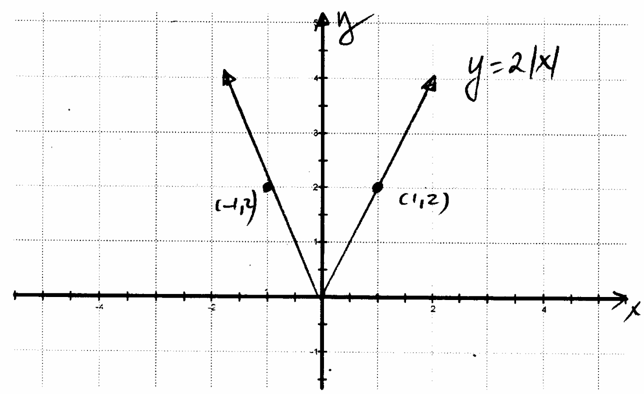
2) Sketch the graph of $y = 2|x-1| + 3$ by showing all transformations applied to the basic function.

Step 1: $y = |x|$



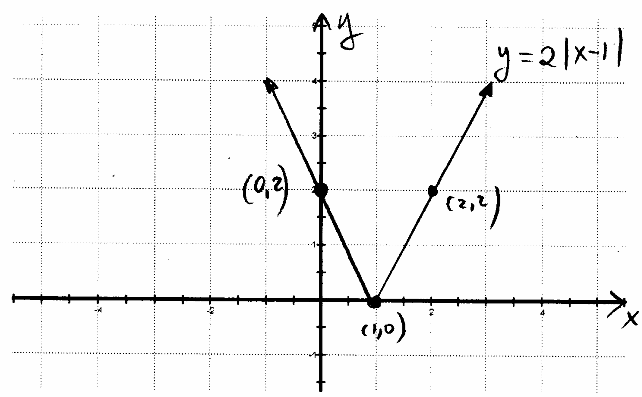

Step 2: $y = 2|x|$

Stretch the previous graph vertically by a factor of 2



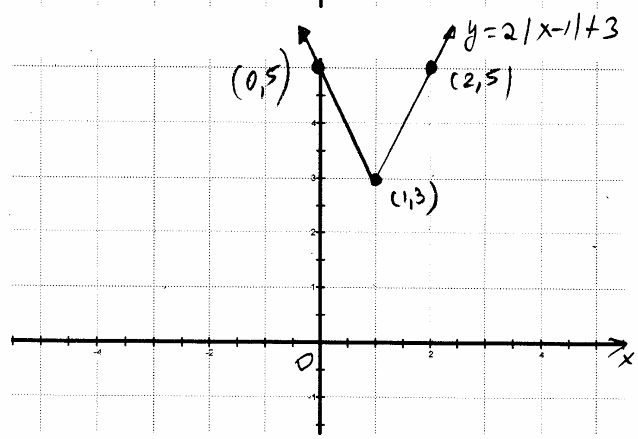

Step 3: $y = 2|x-1|$

shift the previous graph to the right 1 unit.



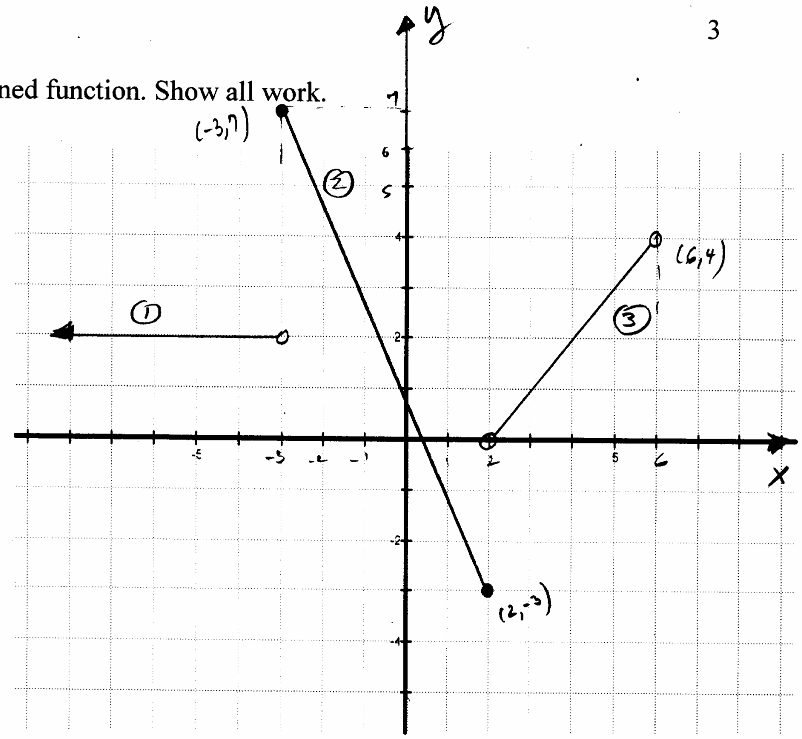

Step 4: $y = 2|x-1| + 3$

shift the previous graph up 3 units.



3) Sketch the graph of the following piece-defined function. Show all work.

$$f(x)=\begin{cases} 2, & \text{if } x<-3 \\ -2x+1, & \text{if } -3\le x\le 2 \\ x-2, & \text{if } 2<x<6 \end{cases}$$




if $x<-3$, $f(x)=2$ ①

if $x\in[-3,2]$, $f(x)=-2x+1$

| x | y |
|---|---|
| -3 | 7 |
| 2 | -3 |

②

if $x\in(2,6)$, $f(x)=x-2$

| x | y |
|---|---|
| 2 | 0 |
| 6 | 4 |

③

Answer the following questions:

a) What is the domain of the function?

$x\in(-\infty,6)$

b) What is the range of the function?

$y\in[-3,7]$

c) Find $f\left(\frac{1}{2}\right)$, $f\left(-\frac{1}{2}\right)$, $f(8)$, and $f(-4)$.

$f(\frac{1}{2})=-2(\frac{1}{2})+1=0$ b/c $x=\frac{1}{2}\in[-3,2]$

$f(-\frac{1}{2})=-2(\frac{-1}{2})+1=2$ b/c $x=\frac{-1}{2}\in[-3,2]$

$f(8)$ is not defined $x=8\notin$ Domain

$f(-4)=2$ b/c $x=-4\in(-\infty,-3)$

d) On what interval(s) is the function increasing ? $x\in(2,6)$

On what interval(s) is the function decreasing ? $x\in[-3,2]$

On what interval(s) is the function constant? $x\in(-\infty,-3)$

4) Let $f(x) = 2x^4 - x^2 + \frac{1}{2}$. Is this function even, odd, or neither? Show appropriate work.

$f(-x) = 2(-x)^4 - (-x)^2 + \frac{1}{2}$
$= 2x^4 - x^2 + \frac{1}{2}$

$f(-x) = f(x) \Rightarrow f$ is an even function.

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5) Let $x^2 + y^2 = 10$.

a) What does this equation represent? Identify the center and radius.

Circle with center $(0,0)$
radius $\sqrt{10}$

b) Does this equation represent a function? Justify your answer.

No, because its graph doesn't pass the vertical line test.
Given $x$, there are two $y$-values

c) Is the graph of the above equation symmetric about the x-axis? Show all work.

yes, b/c it's a circle with center $(0,0)$
if we replace $y$ with $-y$,
$x^2 + (-y)^2 = 10$
$x^2 + y^2 = 10$

d) Is the graph of the above equation symmetric about the y-axis? Show all work.

yes, b/c it's a circle with center $(0,0)$
if we replace $x$ with $-x$,
$(-x)^2 + y^2 = 10$
$x^2 + y^2 = 10$

e) Is the graph of the above equation symmetric about the origin? Show all work.

yes, b/c it's a circle with center $(0,0)$
if we replace $x$ with $-x$ and
$y$ with $-y$.
$(-x)^2 + (-y)^2 = 10$
$x^2 + y^2 = 10$

6) Let $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x}$.

a) What is the domain of f?

$x \in \mathbb{R}$

b) What is the domain of g?

$x \in \mathbb{R} \setminus \{0\}$

c) Calculate $(f \circ g)(x)$ and its domain.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 + 1 = \frac{1}{x^2} + 1$$

$$(f \circ g)(x) = \frac{1}{x^2} + 1$$

$$\text{Domain}_{f \circ g} \quad \begin{cases} x \in \text{Domain of } g \\ \quad \text{and} \\ x \neq 0 \end{cases} \Rightarrow \quad x \in \mathbb{R} \setminus \{0\}$$

$$\text{Domain}_{f \circ g} = \mathbb{R} \setminus \{0\}$$

## PART II

Choose THREE exercises from the following list. Show clearly which exercises you are solving.
You may solve the forth one for extra credit for 7 points. Show clearly which exercise is for extra credit.

1) Solve the following equation. Make sure to check the solutions.

$$\sqrt{2\sqrt{7x+2}} = \sqrt{3x+2}$$

2) Solve the following equation by using an appropriate substitution or by factoring.

$$10x^{-2} + 33x^{-1} - 7 = 0$$

3) Solve the following equation by using an appropriate substitution.

$$(2x-1)^{\frac{2}{3}} + 2(2x-1)^{\frac{1}{3}} - 3 = 0$$

4) Find the domain of the following function.

$$f(x) = \sqrt{x^2 - x - 6}.$$

5) Solve the following inequality.

$$\frac{(x^2 + x - 2)}{(x+1)^2} \geq 0$$

6) Solve in terms of $x$.

$$\begin{cases} 3x + 4y - z = 13 \\ x + y + 2z = 15 \end{cases}$$

① $\sqrt{2\sqrt{7x+2}} = \sqrt{3x+2} \;\Big/^2$

$2\sqrt{7x+2} = 3x+2 \;\Big/^2$

$4(7x+2) = 9x^2 + 12x + 4$

$28x + 8 = 9x^2 + 12x + 4$

$9x^2 - 16x - 4 = 0$

$(9x+2)(x-2) = 0$

$x = -\frac{2}{9}$ or $x = 2$

check $x = -\frac{2}{9}$

$\sqrt{2\sqrt{7(-\frac{2}{9})+2}} \overset{?}{=} \sqrt{3(\frac{-2}{9})+2}$

$\sqrt{2\sqrt{-\frac{14}{9}+2}} \overset{?}{=} \sqrt{-\frac{2}{3}+2}$

$\sqrt{2\sqrt{\frac{4}{9}}} \overset{?}{=} \sqrt{\frac{4}{3}}$

$\sqrt{2\cdot\frac{2}{3}} \overset{?}{=} \frac{2}{\sqrt{3}}$

$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$ true

check $x = 2$

$\sqrt{2\sqrt{16}} \overset{?}{=} \sqrt{8}$

$\sqrt{2\cdot 4} = \sqrt{8}$

true

The solution set is $\{-\frac{2}{9}, 2\}$

(2) $10x^{-2} + 33x^{-1} - 7 = 0$

Let $\boxed{x^{-1} = t}$

Then $10t^2 + 33t - 7 = 0$

$(5t - 1)(2t + 7) = 0$

$\underline{t = \frac{1}{5}}$ OR $\underline{t = \frac{-7}{2}}$

$x^{-1} = \frac{1}{5}$ $\quad$ $x^{-1} = \frac{-7}{2}$

$\frac{1}{x} = \frac{1}{5}$ $\quad$ $\frac{1}{x} = \frac{-7}{2}$

$\boxed{x = 5}$ $\quad$ $\boxed{x = \frac{-2}{7}}$

The solution set is $\{5, \frac{-2}{7}\}$

(3) $(2x-1)^{\frac{2}{3}} + 2(2x-1)^{\frac{1}{3}} - 3 = 0$

Let $\boxed{(2x-1)^{\frac{1}{3}} = t}$

Then $t^2 + 2t - 3 = 0$

$(t+3)(t-1) = 0$

$\underline{t = -3}$ OR $\underline{t = 1}$

$(2x-1)^{\frac{1}{3}} = -3 \,\Big|^3$ $\quad$ $(2x-1)^{\frac{1}{3}} = 1$

$2x - 1 = (-3)^3$ $\quad$ $2x - 1 = 1$

$2x - 1 = -27$ $\quad$ $2x = 2$

$2x = -26$ $\quad$ $\boxed{x = 1}$

$\boxed{x = -13}$.

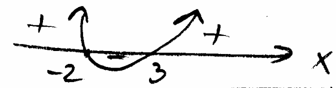
(4) $f(x) = \sqrt{x^2 - x - 6}$

Domain = ?

Condition: $x^2 - x - 6 \geq 0$

$(x-3)(x+2) \geq 0$

parabola opens up




$\boxed{x \in (-\infty, -2] \cup [3, \infty)}$.

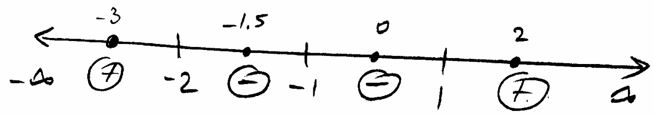
(5) $\dfrac{x^2+x-2}{(x+1)^2} \geq 0$

$\dfrac{(x+2)(x-1)}{(x+1)^2} \geq 0$

| $x$ | $-\infty$ | $-2$ | $-1$ | $1$ | $\infty$ |
|---|---|---|---|---|---|
| $x+2$ | $- \ -$ | $0 \ + \ +$ | $+ \ + \ +$ | | |
| $x-1$ | $- \ -$ | $- \ -$ | $-$ | $0 \ + \ + \ +$ | |
| $(x+1)^2$ | $+ \ +$ | $+$ | $0 \ +$ | $+ \ +$ | |
| $\dfrac{(x+2)(x-1)}{(x+1)^2}$ | $+$ | $0 \ -$ | $\vert \ -$ | $0 \ + \ +$ | |

$$\boxed{x \in (-\infty, -2] \cup [1, \infty)}$$

OR




TP $x=-3$ $\quad \dfrac{(-)(-)}{(+)} = (+)$

TP $x=-1.5$ $\quad \dfrac{(+)(-)}{(+)} = (-)$

TP $x=0$ $\quad \dfrac{(+)(-)}{(+)} = (-)$

TP $x=2$ $\quad \dfrac{(+)(+)}{(+)} = (+)$

$x \in (-\infty, -2] \cup [1, \infty).$

(6) $\begin{cases} 3x + 4y - z = 13 \\ x + y + 2z = 15 \end{cases} \Big/ \cdot 2$

$\begin{cases} 6x + 8y - 2z = 26 \\ x + y + 2z = 15 \end{cases}$

$7x + 9y = 41$

$9y = -7x + 41$

$$\boxed{y = \frac{-7}{9}x + \frac{41}{9}}$$

$\begin{cases} 3x + 4y - z = 13 \\ x + y + 2z = 15 \end{cases} \Big/ \cdot -4$

$\begin{cases} 3x + 4y - z = 13 \\ -4x - 4y - 8z = -60 \end{cases}$

$-x - 9z = -47$

$-x + 47 = 9z$

$$\boxed{z = \frac{-1}{9}x + \frac{47}{9}}$$

There is an infinite number of solutions

$\left(x, \frac{-7}{9}x + \frac{41}{9}, \frac{-1}{9}x + \frac{47}{9}\right), \ x \in \mathbb{R}$