DEFINITION ON SYMMETRY

| Type of symmetry | How to test for symmetry | What the graph looks like | Geometric meaning |
|---|--|---------------------------------|---|
| Symmetry with respect to the <i>x</i> -axis | The equation is unchanged when y is replaced by $-y$. | y (x,y) 0 (x,-y) x | Graph is unchanged when reflected in the <i>x</i> -axis. |
| Symmetry with respect to the y-axis | The equation is unchanged when x is replaced by $-x$. | (-x,y) 0 x | Graph is unchanged when reflected in the y-axis. |
| Symmetry with respect to the origin | The equation is unchanged when x is replaced by $-x$ and y by $-y$. | y (-x,-y) y (x,y) x | Graph is unchanged when rotated 180° about the origin. This is the same as a reflection in the <i>x</i> -axis followed by a reflection in the <i>y</i> -axis. |

| TRANSFORMATION | EQUATION | HOW TO OBTAIN THE GRAPH | WHAT THE GRAPH LOOKS LIKE |
|---|---|---|---|
| Vertical shifts of graphs | y = f(x) + c, $(c > 0)y = f(x) - c$, $(c > 0)$ | Shift graph of $y = f(x)$ upward c units. Shift graph of $y = f(x)$ downward c units. | y y=f(x)+c y=f(x) y=f(x)-c v x |
| Horizontal shifts of graphs | y = f(x-c), (c > 0) y = f(x+c), (c > 0) | Shift graph of $y = f(x)$ to the right <i>c</i> units. Shift graph of $y = f(x)$ to the left <i>c</i> units | y = f(x) $y = f(x)$ $y = f(x)$ $y = f(x-c)$ x |
| Reflecting graphs | y = -f(x) $y = f(-x)$ | Reflect the graph of $y = f(x)$ in the <i>x</i> -axis. Reflect the graph of $y = f(x)$ in the <i>y</i> -axis. | y=f(-x) y=f(x) y=f(x) y=f(x) y=f(x) |
| Vertical stretching and shrinking of graphs | $y = af(x), \qquad (a > 1)$ $y = af(x), \qquad (0 < a < 1)$ | Stretch the graph of $y = f(x)$ vertically by a factor of <i>a</i> . Shrink the graph of $y = f(x)$ vertically by a factor of <i>a</i> . | y y=af(x) $a>1$ y=af(x) $y=af(x)$ $(0$ |
| Horizontal shrinking and stretching of graphs | y = f(ax), (a > 1) y = f(ax), (0 < a < 1) | Shrink the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$. Stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$. | y y=f(ax) (a>1) $y=f(x)0y=f(ax) (1>a>0)$ |

TRANSFORMATIONS OF FUNCTIONS