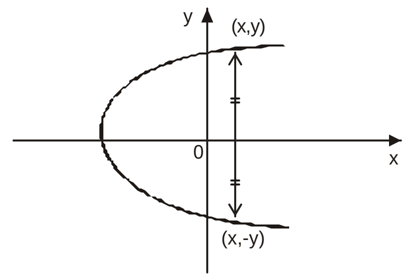
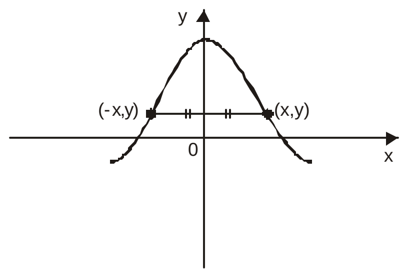
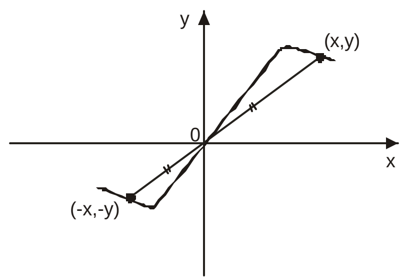
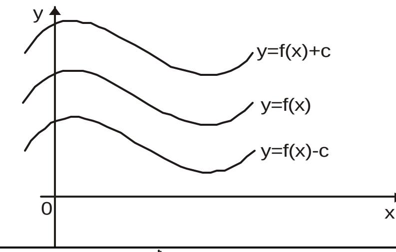
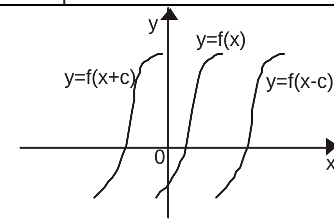
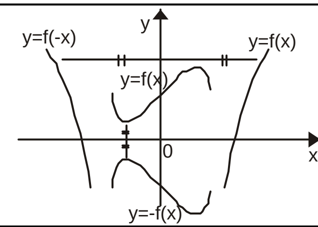
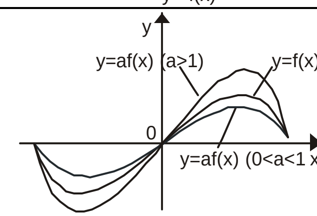
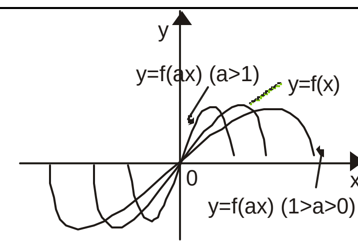
# DEFINITION ON SYMMETRY

| Type of symmetry | How to test for symmetry | What the graph looks like | Geometric meaning |
|---|---|---|---|
| **Symmetry with respect to the *x*-axis** | The equation is unchanged when $y$ is replaced by $-y$ . |   | Graph is unchanged when reflected in the $x$-axis. |
| **Symmetry with respect to the y-axis** | The equation is unchanged when $x$ is replaced by $-x$ . |   | Graph is unchanged when reflected in the $y$-axis. |
| **Symmetry with respect to the origin** | The equation is unchanged when $x$ is replaced by $-x$ and $y$ by $-y$ . |   | Graph is unchanged when rotated $180^o$ about the origin.<br>This is the same as a reflection in the $x$-axis followed by a reflection in the $y$-axis. |

# TRANSFORMATIONS OF FUNCTIONS

| TRANSFORMATION | EQUATION | HOW TO OBTAIN THE GRAPH | WHAT THE GRAPH LOOKS LIKE |
|---|---|---|---|
| Vertical shifts of graphs | $y = f(x) + c, \quad (c > 0)$<br><br>$y = f(x) - c, \quad (c > 0)$ | Shift graph of $y = f(x)$ **upward** $c$ units.<br><br>Shift graph of $y = f(x)$ **downward** $c$ units. |  |
| Horizontal shifts of graphs | $y = f(x - c), \quad (c > 0)$<br><br>$y = f(x + c), \quad (c > 0)$ | Shift graph of $y = f(x)$ to the **right** $c$ units.<br><br>Shift graph of $y = f(x)$ to the **left** $c$ units |  |
| Reflecting graphs | $y = -f(x)$<br><br>$y = f(-x)$ | Reflect the graph of $y = f(x)$ in the ***x*-axis.**<br><br>Reflect the graph of $y = f(x)$ in the ***y*-axis**. |  |
| Vertical stretching and shrinking of graphs | $y = af(x), \quad (a > 1)$<br><br>$y = af(x), \quad (0 < a < 1)$ | Stretch the graph of $y = f(x)$ vertically by a factor of *a*.<br><br>Shrink the graph of $y = f(x)$ vertically by a factor of *a*. |  |
| Horizontal shrinking and stretching of graphs | $y = f(ax), \quad (a > 1)$<br><br>$y = f(ax), \quad (0 < a < 1)$ | Shrink the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$.<br>Stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$. |  |