Section 2.3
Linear Functions. Equations of Lines. Curve Fitting

Linear Equation in Two Variables

Standard form: $ax + by = c$

Slope –Intercept form: $y = mx + b$, where m is the slope of the line, b is the y-intercept

Slope –Point form: $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) a point on the line.

Vertical Line: $x = k$, where k is a constant

Horizontal Line: $y = k$, where k is a constant.

Slope of a Line

$m = \dfrac{change\ in\ y}{change in\ x}$ as we move from one point to another on the line.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$$

The slope m is the rate of change of y with respect to x.

Properties of Lines

Two distinct lines are parallel if and only if they have the same slope or if both are vertical with undefined slope.

$$l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$$

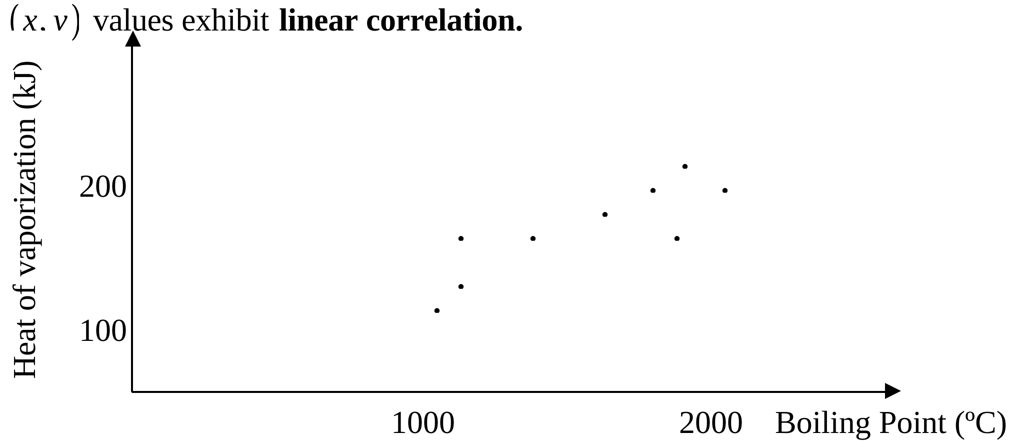
Two lines are perpendicular if and only if the product of their slopes is -1.

$$l_1 \perp l_2 \Leftrightarrow m_1 \cdot m_2 = -1$$

Fitting Linear Functions to Bivariate Data

An equation that relates two variables can be used to find values of one variable from the value of the other. We will consider methods for fitting a linear equation to a collection of data points.

For example, the figure below is called a **scatter-plot.** Each point on a scatter-plot exhibits a pair of measurements about a single event. The points on a scatter-plot may or may not show some sort of a pattern. When the points in a scatter-plot are loosely arranged along some path which rises or falls while moving across the plot, we say that the ***x* and *y* variables are dependent**. When the variables are dependent, it is possible to make estimates of one corresponding to a chosen value of the other. In such cases we say that the variables **correlate** to one another. In our example, although the points do not lie on a straight line, they seem to be clustered around some imaginary line. We say that the paired (x, y) values exhibit **linear correlation.**



Linear regression:

If the data in a scatter-plot are roughly linear, we can estimate the location of an imaginary "line of best fit" that passes as close as possible to the data points. We can use this line to make predictions about the data (when drawing the line that "fits" the data points as best as we can, we try to end up with roughly equal numbers of data points above and below our line).

Error estimates with Linear Functions

Error in a measured value – It is impossible to know the exact value of any error, since this would require exact knowledge of the value which is being estimated. The best that can ever be offered is an estimate of the **maximum error** in any estimate. This depends on the measuring device, and is usually one of the listed specifications of a given device.

Error in a computed value – Maximum error in a computed value is easy to estimate in cases when the computation formula is a linear function.

If y_1 is an unknown value and y is its computed estimate, then $\Delta y = y - y_1$ is the computed error in the estimate of y_1. If x_1 is an unknown value and x is its measured estimate, then $\Delta x = x - x_1$ is the measured error in the estimate of x_1.

Then $y - y_1 = m(x - x_1)$ becomes $\boxed{\Delta y = m \cdot \Delta x}$, where Δy is the computed error / maximum error in the computed y-value, and Δx is the measured error / maximum error in the measured x-value.

Then the formula for $\textit{Maximum Relative Error} = \dfrac{\textit{Maximum Absolute Error}}{\textit{Estimate}} \cdot 100\%$ becomes

$$\boxed{\textit{Maximum Relative Error} = \frac{\Delta y}{y} \cdot 100\%}$$

Application Problems

1. (Problem 44-page 152) A company has a daily fixed cost of $750 and sells bird houses which cost $15 each to manufacture.

   a) Construct a linear function which gives the total cost to produce *x* bird houses.
   b) What is the total cost to produce 50 bird houses? Write this using function notation.
   c) How many bird houses can be manufactured for $2000 ? Write this using function notation.

2. (Problem 45 – page 152) When a given company manufactures 250 units, they incur a total operating cost of $31,000.When they manufacture 425 units, the total operating cost is $33,975. Give the total cost function, with the fixed and marginal costs.

3. (Problem 50 – page 152) If $12,000 is invested at 6% simple interest, give a linear function which gives the value of the investment at time t. When will the value on this investment reach $15,000?

4. (Problem 51 – page 153) Below are yearly precipitation totals (x) paired with annual lowest lake levels (in feet down from a full lake) (y) at Big Bear Lake, in California, for collection of randomly selected years. Create a scatter-plot for these values. Then pick two points which represent the linear trend amongst the data points and determine the equation of the line which contains these points. Then use the equation of this line to predict the annual lowest lake level for a year with 40 inches of precipitation.

x=Precipitation (inches)	*y*=Lowest Lake Level (feet below full)
15.02	13.98
30.62	9.89
24.82	7.38
50.40	1.22
27.00	5.67
41.04	4.57
49.00	2.48
31.78	3.78
22.20	14.45
17.32	12.17
24.18	9.48
27.49	7.00
35.16	5.63
22.40	6.48

5. (Problem 55 – page 154) Suppose a particular medication is to be administered to a patient at a rate of 250 mg per 100 lb. If a patient's weight is estimated at 170 lb with a maximum error of 5 lb, what is the patient's dosage and the corresponding maximum error and maximum relative error in the computed dosage?

6. (Problem 57 – page 155) A particular brand of rice costs $1.25 per pound. A buyer purchases 4.6 lb of rice with a maximum error of 0.24 lb. Give the corresponding price and the maximum error and maximum relative error in the price.