1.3 Numeracy

1.3.2 Types of Numbers

Definition **A set** is a collection of objects (elements).

The Set of Natural Numbers $\mathbb{N}$ — $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

The Set of Whole Numbers W — $W = \{0, 1, 2, 3, 4, 5, \ldots\}$

The Set of Integers $\mathbb{Z}$ — $\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

The Set of Rational Numbers $\mathbb{Q}$ — $\mathbb{Q} = \left\{\frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0\right\}$

The Set of Irrational Numbers J — $J = \{x | x \text{ is not rational}\}$

Examples: $\sqrt{2}, -\sqrt{5}, \pi$

The Set of Real Numbers $\mathbb{R}$ — $\mathbb{R} = \{x \mid x \text{ is rational or } x \text{ is irrational}\}$

1.3.3. Rounding and Significant Figures

Definition **Rules for Significant Figures**

The following rules explain which figures are significant in a measured value.

1. All non-zero digits are significant.
   3.518 has 4 significant figures
2. Zeros between non-zero digits are significant.
   3.0518 has 5 significant figures
3. Leading zeros are not significant.
   0.0035 has 2 significant figures
4. Trailing zeros to the right of the decimal point are significant.
   3.5180 has 5 significant figures
   0.02000 has 4 significant figures
5. Trailing zeros to the left of the decimal point are questionable.
   a. If the measurement is exact, the trailing zeros are significant.
   It is known there are exactly 500 people at a party (3 significant digits)
   b. Otherwise, the trailing zeros are insignificant.

We estimate that 500 people attended a party (1 significant digits)

Exercise 1 Give the number of significant figures in the following computations:

0.0043080 – 5 significant figures

4.270×10^6 – 4 significant fig.

2300.0 – 5 significant figures

2500 is accurate to the nearest hundred
2 significant fig.

Rules for Rounding Computational Results

1. When adding or subtracting: round to the fewest number of digits after the decimal that appear in any of the given values

2. When multiplying or dividing: the result has the same number of significant digits as the given value with fewest significant digits

3. When applying other mathematical operations (radicals, logarithms, etc): the result has the same number of significant digits as the given value with fewest significant digits

4. In all cases, exact values that are not approximated have an infinite number of significant figures.

Note: Rounding rules only apply when input values are known to be rounded. When that is not the case, the rounding is somewhat arbitrary.

Exercise 2 Perform the operations, rounding to the correct number of significant figures, assuming all whole numbers are exact.

$32.7 + 6.01 - 0.0047 = 38.7053$
≈ 38.7

$\dfrac{\dfrac{3.7-4.5}{5.2}}{\sqrt{46}} = -1.0434354 \approx -1.0$

$21032.25 \cdot 0.00413 \div 0.210 =$
$= 413.63425$
≈ 414

$\dfrac{(14-1)1.91^2 + (36-1)7.13^2}{14+36-2} = 38.0566$
≈ 38.1

$\sqrt{146.2} = 12.091319$
≈ 12.09

$\dfrac{0.139-0.132}{\sqrt{\dfrac{0.015^2}{26} + \dfrac{0.011^2}{38}}} = 2.0345006$
≈ 2.0

Exercise 3 (#16 page 36) The standard deviation formula from statistics is as follows:

$$s=\sqrt{\frac{\sum x^2-\frac{1}{n}\left(\sum x\right)^2}{n-1}}.$$ Evaluate this formula using $\sum x = 50.58$, $\sum x^2 = 502.1444$, and $n=7$, where n is exact. Round to the correct number of significant digits.

$$S=\sqrt{\frac{502.1444-\frac{1}{7}(50.58)^2}{7-1}} = 4.7726263$$
$$\approx 4.773$$

Exercise 4 (#18 page 36) One application in statistics uses a formula for degrees of freedom, V, using the formula

$$V=\frac{(V_1+V_2)^2}{\frac{V_1^2}{n_1-1}+\frac{V_2^2}{n_2-1}},$$ where $V_1=\frac{s_1^2}{n_1}$ and $V_2=\frac{s_2^2}{n_2}$. Evaluate V where

$s_1=3.19, n_1=53, s_2=6.73, n_2=41$, where n_1 and n_2 are exact, and round appropriately.

$$V=\frac{\left(\frac{3.19^2}{53}+\frac{6.73^2}{41}\right)^2}{\frac{\left(\frac{3.19^2}{53}\right)^2}{52}+\frac{\left(\frac{6.73^2}{41}\right)^2}{40}} = 53.8610 \approx 53.9$$

1.3.4 Scientific Notation

Definition A number is written in scientific notation if it expressed in the form

$$a\cdot 10^n$$

where $|a|\in[1,10)$ and $n\in Z$.

Exercise 5 Express in scientific notation:

$234.123 = 2.34123 \times 10^2$

$0.002137 = 2.137 \times 10^{-3}$

$1025.011 = 1.025011 \times 10^3$

$-0.000102 = -1.02 \times 10^{-4}$

Note: When computing with scientific notation or without, the number of significant figures is the same in either representation. For example, 127.890 should be converted to 1.27890×10^2.

Exercise 6 Express in scientific notation, rounding to 4 significant figures:
(#6 page 35)

$$\frac{2,753,\cancel{111}.4}{\cancel{0}.0142} = 193,881,084.5$$
$= 1.938810845 \times 10^8$ (scientific notation)
$\approx 1.939 \times 10^8$ (4 significant figures)

Exercise 7 Express in scientific notation, rounding to 3 significant digits:
(#8 page 35)

$120 \cdot (0.44)^{12} \cdot (0.56)^{20-12} = 0.0000611107$
$= 6.11107 \times 10^{-5}$ (scientific not.)
$\approx 6.11 \times 10^{-5}$ (3 significant fig.)

1.3.8 Error

$$\text{Maximum Relative Error} = \frac{\textit{Maximum Absolute Error}}{\textit{Estimate}} \cdot 100\%$$

Exercise 8 (Example 1.3.29 page 33) Originally the speed of light was estimated by averaging a collection of experimentally measured values. One of the earliest estimates was 299,852,400 meters per second, with an estimated maximum absolute error of 20,400 meters per second. Find the maximum relative error.

Given: $\begin{cases} \text{estimate} = 299,852,400 \ \frac{m}{s} \\ \text{max. abs. error} = 20,400 \ \frac{m}{s} \end{cases}$

Then,
$$\text{max. rel. error} = \frac{20,400 \ \cancel{\frac{m}{s}}}{299,852,400 \ \cancel{\frac{m}{s}}} \cdot 100\% = 0.0068\%$$

Exercise 9 (#27 page 37) Suppose the president is estimate to have a 59% approval rating, but this estimate has a maximum error of 4 percentage points. What is the maximum relative error?

Given: $\begin{cases} \text{estimate} = 59\% \\ \text{max abs. error} = 4\% \end{cases}$

Then,
$$\text{max. rel. error} = \frac{4 \cancel{\%}}{59 \cancel{\%}} \cdot 100\%$$
$$= \frac{4}{59} \cdot 100\% = 6.77966\%$$
$$\approx 6.8\%$$