Chapter 6
Polynomials, Polynomial Functions, and Factoring

Factoring a polynomial

1. GCF Factor out the greatest common factor (if any).

2. Special products

Two terms

$$a^2 - b^2 = (a-b)(a+b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Three terms

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

3. Factoring techniques to factor out a trinomial $ax^2 + bx + c$

$a = 1$

$x^2 + bx + c = (x + \square)(x + \square)$

product = c
sum = b

$a \neq 1$

split the middle term bx

$ax^2 + bx + c = ax^2 + \square x + \square x + c$

product = ac
sum = b
then factor by grouping

4. If more than four term, factor by grouping.

3. Simplify: $\left(y^n+2\right)\left(y^n-2\right)-\left(y^n-3\right)^2$

4. Factor each polynomials completely:

a) $2y^7(3x-1)^5-7y^6(3x-1)^4$

b) $ax+ay+az-bx-by-bz+cx+cy+cz$

c) $x^ny^n+3x^n+y^n+3$

d) $x^{4n}+x^{2n}+x^{3n}$

e) $3x^{3m}y^m-6x^{2m}y^{2m}$

f) $15x^3-25x^2+10x$

g) $24x^2+3xy-27y^2$

h) $4x^3y^5+24x^2y^5-64xy^5$

i) $(3x-y)\left(x^2-2\right)+\left(x^2-2\right)$

j) $(a+b)^2+a+b$

k) $15x^{2n}-25x^n$

5. Factor by introducing an appropriate substitution.

a) $2x^4-x^2-3$

b) $2x^6+11x^3+15$

c) $3(x-2)^2-5(x-2)-2$

d) $9x^{2n}+x^n-8$

e) $a^{2n+2}-a^{n+2}-6a^2$

6. Factor completely.

a) $y^2-12y+20$

b) $x^2-9xy+14y^2$

c) $3x^2+3x-18$

d) $4y^3+12y^2-72y$

e) $3x^2+8x+5$

f) $10a^2+19a+6$

g) $6x^2+19x+15$

h) $15x^2+11xy-14y^2$

i) $15y^5-2y^4-y^3$

j) $2d^{n+2}-5d^{n+1}+3d^n$

k) $mp-np-m+n$

l) $0.04x^2+0.12x+0.09$

m) $acx^2-bcx+adx-bd$

n) $x^2-\frac{6}{25}+\frac{1}{5}x$

o) $x^3-12-3x^2+4x$

p) $8x^4-\frac{x}{8}$

q) $x^5-x^3+27x^2-27$

r) $x^ny^n-x^n+2y^n-2$

7. If $(fg)(x)=3x^2-22x+39$, find *f* and *g*.

9. Factor completely:

a) $4a^3c^2-16ax^2y^2$

b) $8x^2+8y^2$

c) $1-81x^4$

d) x^3-6x^2-x+6

e) $16x^2-40xy+25y^2$

f) $x^2-8xy+64y^2$

g) $x^2-6x+9-y^2$

h) $25x^2-20x+4-81y^2$

i) $125x^3-8$

j) $216x-x^4$

k) x^9+1

l) $x^3+(x+y)^3$

m) $4x^2-9$

n) $1+x^2y^2$

o) $2x^3-8x$

p) x^2+4x+4

r) $x^2-6x+9-y^2$

10. Solve each equation by factoring.

a) $x^2-4x=45$ (A: -5,9)

b) $x^2=8x$ (A: 0,8)

c) $(x-3)(x+8)=-30$ (A: -3, -2)

d) $\frac{x^2}{4}-\frac{5x}{2}+6=0$ (A: 4, 6)

e) $(2x-1)\left(3x+\frac{1}{2}\right)(x-1)^2=0$

f) $x^3+4x^2-25x-100=0$ (A: -5, -4, 5)

g) $3x^4-48x^2=0$ (A: -4, 0, 4)

h) $x(x+1)^3-42(x+1)^2=0$ (A: -7, -1, 6)

i) $-7x\left[x(3x-2)-8\right](25x^2-40x+16)=0$ (A: -4/3, 4/5, 0, 2).

j) $\left|x^2+2x-36\right|=12$ (A: -8, -6, 4, 6)

11. Find all numbers satisfying the given conditions:
a) If 5 is subtracted from 3 times the number, the result is the square of 1 less than the number. (A: 2, 3)
b) The product of the number decreased by 1 and increased by 4 is 24. (A:-7, 4)

12. a) Write a quadratic equation in standard form whose solutions are -3 and 7.
b) Write a quadratic equation in standard form with integer coefficients whose solutions are -1/2 and 3/5.

13. Let $f(x) = x^3 + 4x^2 - x + 6$. Find all values of *c* such that $f(c) = 10$. (A: -4,-1,1)

Polynomial Equations and Their Applications

1. James Bond stands on top of a 240-foot building and throws a film canister upward to a fellow agent in a helicopter 16 feet above the building. The height of the film above the ground *t* seconds later is given by the formula $h = -16t^2 + 32t + 240$ where *h* is in feet.

a) Calculate $h(0)$ and $h(1)$. What is their meaning in this context?
b) How long will it take the film canister to reach the agent in the helicopter? (A: 1 sec)
c) If the agent misses the canister, when will it pass James Bond on the way down? (A: 2 sec)
d) How long will it take to hit the ground? (A: 5 sec)

2. A rectangular parking lot has a length that is 3 yards greater than the width. The area of the parking lot is 180 square yards. Find the length and width. (A: 15 yd ;12 yd)

3. As part of a landscaping project, you put in a flower bed measuring 20 feet by 30 feet. To finish off the project, you are putting in a uniform border of pine bark around the outside of the rectangular garden. You have enough pine bark to cover 336 square feet. How wide should the border be? (A: 3 ft)

4. The size of a rectangular computer monitor screen is given by the length of its diagonal. If the length of the screen should be 3 inches greater than its width, what are the dimensions of a 15-inch monitor? (A: 9in by 12 in)

5. A tree is supported by a wire anchored in the ground 15 feet from its base. The wire is 4 feet longer than the height that it reaches on the tree. Find the length of the wire. (A: 30 1/8 ft)

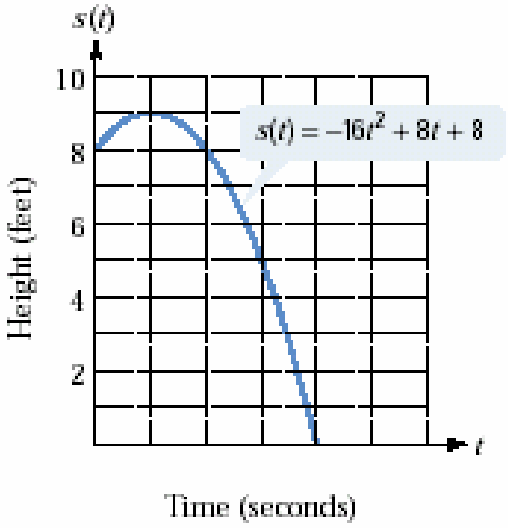
6. The height , *h*, of a baseball *t* seconds after being hit is given by $h = -16t^2 + 64t + 4$. a)When will the baseball reach a height of 64? b) What was the initial height of the ball? c)When will be ball reach the ground? (A: a) 3/2, 5/2 sec)

7. A machine produces open boxes using square sheets of metal. The machine cuts equal-sized squares measuring 3 inches on a side from the corners and then shapes the metal into an open box by turning up the sides. If each box must have a volume of 75 cubic inches, find the length and width of the open box. (A: 5 in)

8. A car traveling at 50 feet per second (about 34 mi per hour) can stop in 2.5 seconds after applying the brakes hard. The distance the car travels in feet, *t* seconds after applying the brakes is $d = 50t - 10t^2$. How long does it take the car to travel 40ft? (A: 1 second)

9. A lot is in the shape of a right triangle. The longer leg of the triangle is 20 yards longer than twice the length of the shorter leg. The hypotenuse is 30 yards longer than twice the length of the shorter leg. What are the lengths of the three sides? (A: 50, 120, and 130)

10.
A gymnast dismounts the uneven parallel bars at a height of 8 feet with an initial upward velocity of 8 feet per second. The function $s(t) = -16t^2 + 8t + 8$ describes the height of the gymnast's feet above ground, $s(t)$, in feet, t seconds after dismounting. The graph of the function is shown, with unlabeled tick marks along the horizontal axis.



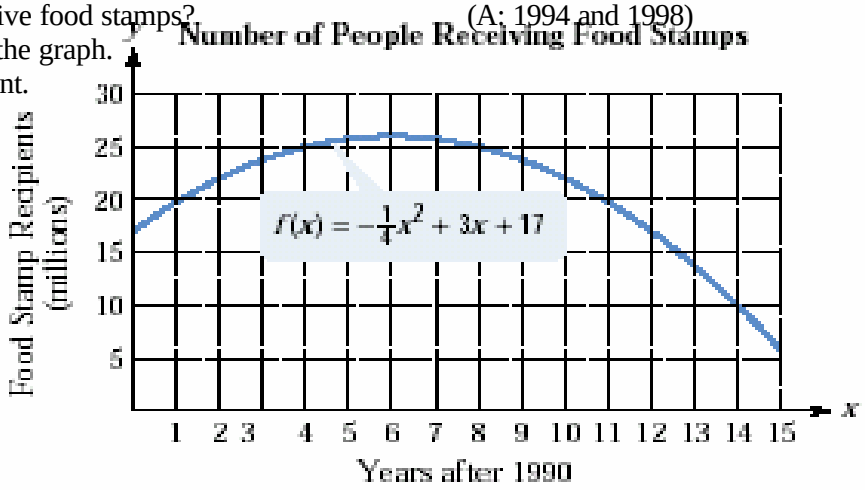
a) How long will it take the gymnast to reach the ground? Use this information to provide a number on each tick mark along the horizontal axis in the figure.

b) When will the gymnast be 8 feet above the ground? Identify the solution(s) as one or more points on the graph.

11. The Food Stamp Program is America's first line of defense against hunger for millions of families. Over half of all participants are children; one out of six is a low-income older adult. The next questions involve the number of participants in the program from 1990 through 2002.

The function $f(x) = -\frac{1}{4}x^2 + 3x + 17$ models the number of people, $f(x)$, in millions, receiving food stamps x years after 1990.

a) In which year did 26 million people receive food stamps?
b) In which years did 25 million people receive food stamps? (A: 1994 and 1998)
c) Identify your solution in (a) as a point on the graph. Describe what is significant about this point.
d) Identify your solution in (b) as one or more points on the graph. Then describe the trend shown by the graph.
e) How many people received food stamps in 1996? (A: 26 million)



Answers

#3 $6y^n - 13$; **#4**d) $x^{2n}\left(x^{2n} + x^n + 1\right)$;e) $3x^{2m}y^m\left(x^m - 2y^m\right)$;f) $5x(3x-2)(x-1)$;g) $3(8x+9y)(x-y)$;
h) $4xy^5(x+8)(x-2)$. **#5** c) $(3x-5)(x-4)$;d) $\left(9x^2-8\right)\left(x^2+1\right)$;e) $a^2\left(a^n-3\right)\left(a^n+2\right)$ **#6** e) $(3x+5)(x+1)$
d) $4y(y+6)(y-3)$; f) $(5x+2)(2x+3)$; i) $y^3(5y+1)(3y-1)$; j) g) $(2x+3)(3x+5)$; h) $(5x+7y)(3x-2y)$;
j) $d^n(2d-3)(d-1)$;l) $(0.2x+0.3)^2$;m) $(ax-b)(cx+d)$;n) $\left(x+\frac{3}{5}\right)\left(x-\frac{2}{5}\right)$; q) $(x+1)(x-1)(x+3)\left(x^2-3x+9\right)$
#9 a) $4a(ac-2xy)(ac+2xy)$; f) prime; g) $(x-3+y)(x-3-y)$;
h) $(5x-2-9y)(5x-2+9y)$;j) $x(x-6)\left(36+6x+x^2\right)$; l) $(2x+y)\left(x^2+xy+y^2\right)$.