

REVIEW

Chapter 1 – The Real Number System

In class work: Solve all exercises.

(Sections 1.1 & 1.2)

Definition A **set** is a collection of objects (elements).

The Set of Natural Numbers \mathbb{N}

$$\mathbb{N} = \{ 1, 2, 3, 4, 5, \dots \}$$

The Set of Whole Numbers \mathbb{W}

$$\mathbb{W} = \{ 0, 1, 2, 3, 4, 5, \dots \} \quad \mathbb{N} \subset \mathbb{W}$$

The Set of Integers \mathbb{Z}

$$\mathbb{Z} = \{ \dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots \}$$

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z}$$

The Set of Rational Numbers \mathbb{Q}

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0 \right\} \quad \mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q}$$

The Set of Irrational Numbers

Examples: $\sqrt{2}, -\sqrt{5}, p$

The Set of Real Numbers \mathbb{R}

$$\mathbb{R} = \{ x \mid x \text{ is rational or } x \text{ is irrational} \}$$

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Mathematical Symbols

SYMBOL	MEANING	EXAMPLES
=	is equal to	
≠	is not equal to	
∈	belongs to (about an element)	
∉	it doesn't belong to	
<	is less than	
≤	is less than or equal to	
>	is greater than	
≥	is greater than or equal to	
∀	any	

Properties of Real Numbers

PROPERTIES	ADDITION +	MULTIPLICATION •
COMMUTATIVITY	$a + b = b + a, \quad \forall a, b \in \mathbb{R}$	$ab = ba \quad \forall a, b \in \mathbb{R}$
ASSOCIATIVITY	$(a + b) + c = a + (b + c), \forall a, b, c \in \mathbb{R}$	$(ab)c = a(bc), \quad \forall a, b, c \in \mathbb{R}$
IDENTITY ELEMENT	0 $a + 0 = 0 + a = a, \forall a \in \mathbb{R}$	1 $a \cdot 1 = 1 \cdot a = a, \forall a \in \mathbb{R}$
INVERSE ELEMENT	$\forall a \in \mathbb{R}$, there is $-a \in \mathbb{R}$ such that $a + (-a) = (-a) + a = 0$	$\forall a \in \mathbb{R}, a \neq 0$, there is $\frac{1}{a} \in \mathbb{R}$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$
DISTRIBUTIVITY	$a(b + c) = ab + ac$ <div style="display: flex; justify-content: center; align-items: center; gap: 20px;"> <div style="text-align: center;"> $\xrightarrow{\hspace{10em}}$ multiply out (remove parentheses) </div> <div style="text-align: center;"> $\xleftarrow{\hspace{10em}}$ factor out the common factor </div> </div>	

Exercise #1 Find the opposite and the reciprocal (if any) of each number:

The Number	Its Opposite	Its Reciprocal

The Double Negative Rule

$$-(-a) = a$$

(Section 1.2)

The Absolute Value of a Number

Definition (1) The **absolute value of a number** is the distance between the number and 0 (the origin) on the number line.

$$|a| = \text{dist}(a, 0)$$

Property $|a| \geq 0, \quad \forall a \in \mathbb{R}$

Definition (2)
$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Properties (1) $|ab| = |a| \cdot |b|, \quad \forall a, b \in \mathbb{R}$

(2) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}, \quad \forall a, b \in \mathbb{R}, b \neq 0$

Note: $|a+b| \neq |a| + |b|$ Example: _____

$|a-b| \neq |a| - |b|$ Example: _____

Exercise #2 Simplify the following:

- | | |
|--------------|----------------|
| a) $ -7 =$ | c) $- -7 =$ |
| b) $-(-7) =$ | d) $- (-7) =$ |

Exercise #3 Simplify the following:

- a) $(-5)^2 - 3^2 + |10 - 2 \cdot 3|$ (A: 20)
- b) $\frac{(-4)^2 - |1 - 2^3|}{-(-2)^3 + (-1)^{125}}$ (A: $\frac{9}{7}$)
- c) $\frac{9[4 - (1+6)] - (3-9)^2}{5 + \frac{12}{5 - \frac{6}{2+1}}}$ (A: -7)

Exercise #4 Evaluate the following expressions if $x = 2, y = -3, z = -1$:

- | | |
|--|-----------------------------|
| a) $\frac{3y^2 - x^2 + 1}{y z }$ (A: -8) | b) $yz^3 - (xy)^3$ (A: 219) |
|--|-----------------------------|

(Section 1.6)

Properties of Integral Exponents

Definition If $n \in \mathbb{N}$, then $a^n = a \cdot a \cdot \dots \cdot a$
 n times
 a is called **base** and n is called **power (exponent)**.

PROPERTY		EXAMPLES
The Product Rule	$a^m \cdot a^n = a^{m+n}$	
The Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$	
The Zero-Exponent Rule	$a^0 = 1, \forall a \neq 0$	
The Negative-Exponent Rule	$a^{-n} = \frac{1}{a^n}$	
The Power Rule	$(a^m)^n = a^{m \cdot n}$	
Products to Power	$(ab)^n = a^n \cdot b^n$	
Quotients to Power	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

Exercise #5 Simplify the following expressions:

- a) $x - 5[x - 5(x - 5)]$ (A: $21x - 125$)
- b) $x^2y(xy - x) - 7xy(x^2y - x^2)$ (A: $-6x^3y^2 + 6x^3y$)
- c) $(-8xy)(x^5y^4)(-4xy)$ (A: $32x^7y^6$)
- d) $x[2x^2 + x(x - 3(x - 1))]$ (A: $3x^2$)
- e) $2(x - 1)(3x + 2) - 5(2 - x)(2x + 3)$

Exercise #6 Simplify each expression .

Write answers without using parentheses or negative exponents.

- a) $\frac{y^2}{yy^{-2}}$ (A: y^3)
- b) $\left(\frac{a^2b^{-1}}{4a^3b^{-2}}\right)^{-3}$ (A: $\frac{64a^3}{b^3}$)
- c) $\frac{a^0 + b^0}{2(a + b)^0}$ (A: 1)
- d) $\left(\frac{-2a^{-4}b^3c^{-1}}{3a^{-2}b^{-5}c^{-2}}\right)^{-4}$ (A: $\frac{81a^8}{16b^{32}c^4}$)
- e) $\left(\frac{2x^{-4}y}{x^5y^5}\right)^{-3} \left(\frac{4x^{-2}y^0}{x^7y^2}\right)^2$ (A: $2x^9y^8$)
- f) $\frac{24x^2y^{13}}{-2x^5y^{-2}}$ (A: $-\frac{12y^{15}}{x^3}$)
- g) $(-4x^{-4}y^5)^{-2}(-2x^5y^{-6})$ (A: $-\frac{x^{13}}{8y^{16}}$)

(Sections 1.4 & 1.5)

Linear Equations

Definition An **equation** is a mathematical statement that two algebraic expressions are equal.

Examples:

Types of Equations

- (1) **IDENTITY** = an equation which is always **true** regardless of the value of the variable.

Examples: $3 = 3$

$$x + 1 = x + 1$$

- (2) **CONTRADICTION** = an equation which is always **false** regardless of the value of the variable.
(**INCONSISTENT**)

Examples: $5 = 7$

$$x + 2 = x + 4$$

- (3) **CONDITIONAL** = an equation whose truth or falsehood depends on the value of the variable.

Examples: $x + 2 = 5$

Exercise #10 Determine the type of each of the following equations:

- a) $2(x - 3) = 2x - 3$
- b) $5(x + 2) = 5x + 10$
- c) $3(w + 1) = w + 3$

Definition A **solution** of an equation is the value of the variable that **satisfies** the equation.

Definition The process of finding the values that satisfy an equation is called **solving the equation**.

Exercise #11 Determine which of the listed values satisfies the given equation:

a) $2x+3=6$, $x=0$, $x=\frac{3}{2}$

b) $6-2w=10-3w$, $w=-4$, $w=1$

Properties of Equality

$$\text{If } a = b, \text{ then } \left\{ \begin{array}{l} a + c = b + c, \forall c \in \mathbb{R} \\ a - c = b - c, \forall c \in \mathbb{R} \\ ac = bc, \forall c \in \mathbb{R} \\ \frac{a}{c} = \frac{b}{c}, \forall c \neq 0 \end{array} \right.$$

Exercise #12 Solve the following equations .

Objective 1: Solve an equation containing fractions.

1. $\frac{3}{4}z - \frac{1}{4} = \frac{3}{4}$

2. $\frac{4}{5}y - \frac{1}{5} = \frac{3}{5}$

3. $\frac{x+4}{2} + \frac{x+1}{4} = 3$

4. $\frac{w+3}{6} - \frac{w+4}{2} = 2$

5. $\frac{2}{3}(v-4) = 2$

6. $\frac{3}{4}(u-6) = 2$

7. $\frac{5}{3}(t-1) = \frac{4}{5}(2t+1) + \frac{2}{3}$

8. $\frac{4}{5}(s+2) = \frac{1}{2} + \frac{5}{6}(s+3)$

9. $\frac{3(n-2)}{5} = \frac{3n+6}{6}$

Objective #2: Solve an equation containing decimals.

9. $0.8q - 3.2 = 1.6$

10. $2.3r - 4.7 = 4.5$

11. $2.3s + 4.7s = 4.9$

12. $5.1m + 2.3m = 2.96$

13. $0.4(0.2n - 0.3) = 0.01$

14. $0.8(0.3p - 0.5) = 0.8$

15. $0.8q - 0.3(210 - q) = 80$

16. $0.3r + 1.2(20) = 0.8(r + 20)$

Answers Objective #1: 1) $4/3$; 2) 1; 3) 1; 4) $-21/2$; 5) 7; 6) $26/3$; 7) 47; 8) -42; 9) 22

Answers Objective #2: 9) 6; 10) 4; 11) .7; 12) .4; 13) $13/8$; 14) 5; 15) 130; 16) 16

Answers Objective #3: 17) 40; 18) 30; 19) 8; 20) 60; 21) 40; 22) 40; 23) 3

Answers Mixed Practice: 25) $2/11$; 26) $5/7$; 27) -2; 28) -12; 29) $-1/10$; 30) $-2/9$; 31) $-3/10$; 32) $43/12$; 33) 12; 34) $1/12$; 35) 16; 36) 6.175; 37) $-21/19$

Objective #3: Solve an equation containing percents.

17. $15\%q = 6$

18. $30\%r = 9$

19. $50\%s + s = 12$

20. $75\%t + t = 105$

21. $20\%u + 25\%u = 18$

22. $50\%t + 20\%(90 - t) = 30$

23. $20\% + 40\%(25 - s) = 9$

MIXED PRACTICE: Solve each equation.

25. $\frac{1}{4}m + \frac{2}{3}m = \frac{1}{6}$

26. $\frac{1}{3}n + \frac{1}{4}n = \frac{5}{12}$

27. $\frac{3p+2}{2} - \frac{p-5}{3} = \frac{1}{3}$

28. $\frac{q-3}{2} - \frac{4q-1}{6} = \frac{2}{3}$

29. $\frac{3}{4}(2r-5) - \frac{2}{3}(r-6) = \frac{1}{6}$

30. $\frac{3}{4}(s-2) - \frac{3}{5}(2s-3) = \frac{1}{5}$

31. $\frac{2}{5} = \frac{2t+3}{6}$

32. $\frac{5}{6} = \frac{2u-3}{5}$

33. $\frac{v-3}{3} - \frac{v-2}{2} = \frac{4-v}{4}$

34. $\frac{2m-1}{2} - \frac{3m-1}{3} = \frac{4m-1}{4}$

35. $5 = 0.5(q-10) + 2$

36. $0.5r = 0.14 + 0.3(16 - r)$

37. $9(a+5) - 10(1-a) = 14$

38. Evaluate $x^2 - (xy - y)$ for x satisfying $\frac{3(x+3)}{5} = 2x + 6$ and y satisfying $-2y - 10 = 5y + 18$.

Exercise #13 Solve each formula for the specified variable:

a) $v = k + gt$, for t

$$\left(A: t = \frac{v-k}{g} \right)$$

b) $S = 3pd + pa$, for d

$$\left(A: d = \frac{S - pa}{3p} \right)$$

c) $A = P(1 + rt)$, for r

$$\left(A: r = \frac{A - p}{pt} \right)$$

d) $A = 2w^2 + 4lw$, for l

$$\left(A: l = \frac{A - 2w^2}{4w} \right)$$

e) $A = \frac{1}{2}h(a+b)$ for a $\left(A: a = \frac{2A}{h} - b \right)$

f) $A = 2lw + 2lh + 2wh$ for l

$$\left(A: l = \frac{A - 2wh}{2(w+h)} \right)$$