# Section 7.8 – Variation

## Direct Variation

Two variables are **directly proportional** (or just **proportional**) if the ratios of their corresponding values are always equal.

Example: The price of gasoline as a function of the number of gallons purchased.

The ratio $\dfrac{\textit{total price}}{\textit{number of gallons}}$, or price per gallon, is the same no matter how many gallons you buy. Thus, we say that the total cost is directly proportional to the number of gallons purchased.

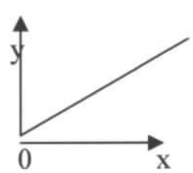
**Direct variation** — ***y* varies directly with *x*** if $\boxed{y = kx}$, where $k$ is a positive constant called the constant of variation.

In general, ***y* varies directly with a power of *x*** if $\boxed{y = kx^n}$, where $k$ and $n$ are positive numbers.

Note: The graph of a direct variation function passes through the origin (0,0).

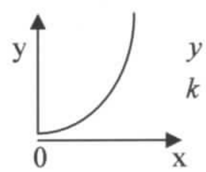
In the case of $y = kx$, $k$ is just the slope of the line, so it tells us how rapidly the graph increases.

In any example of direct variation, as the independent variable increases through positive values, the dependent variable increases also. Thus, direct variation is an example of an increasing function.




$y = kx$

$k > 0$




$y = kx^n$

$k > 0$

Note: " Vary directly" means exactly the same thing as "are directly proportional". The two phrases are interchangeable.

Example: The circumference of a circle varies directly with its radius, because $C = 2\pi r$. The constant of variation is $2\pi$, or about 6.28.

**Exercise #1** If an object is dropped from a great height, say, off the rim of the Grand Canyon, its speed, $v$, varies directly with the time, $t$, the object has been falling. A rock kicked off the edge of the Canyon is falling at a speed of 39.2 meters per second when it passes a lizard on a ledge 4 seconds later.

a) Express $v$ as a function of $t$.

$V = kt$

$V = 39.2 \frac{m}{s}$ when $t = 4s$ $\Big\}$ $\Rightarrow$ $39.2 \frac{m}{s} = k \cdot (4s)$

$k = \dfrac{39.2\ m}{4\ s^2} = 9.8 \frac{m}{s^2}$

So $\boxed{V = 9.8t}$

b) What is the speed of the rock after it has fallen for 6 seconds?

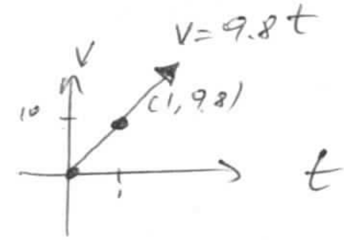
$V = 9.8t$

if $t = 6s$ $\Big\}$ $\Rightarrow$ $V = 9.8(6)$

$V = 58.8\ m/s$

c) Sketch a graph of $v(t)$.

| t | V |
|---|---|
| 0 | 0 |
| 1 | 9.8 |



**Inverse Variation**

Example: How long does it take to travel a distance of 600 miles?
The answer depends on the average speed at which you travel. If you are on a bicycle trip, your average speed might be 15 miles per hour, so your traveling time will be

$$T = \frac{D}{R} = \frac{600}{15} = 40 \text{ hours.}$$

If you are driving a car, you might average 50 miles per hour, so your travel time is then

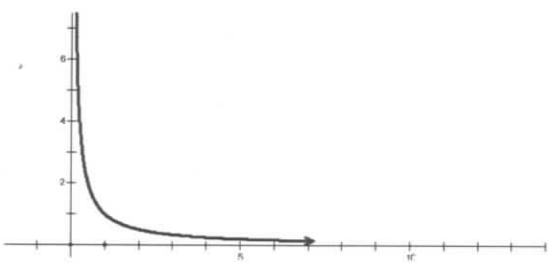
$$T = \frac{D}{R} = \frac{600}{50} = 12 \text{ hours.}$$

You can see that higher average speeds, the travel time is shorter. In other words, the time needed for a 600-mile journey is a decreasing function of average speed. In fact, a formula for the function is $T(R) = \frac{600}{R}$. This is an example of inverse variation.

**Inverse variation** ***y*** **varies inversely with** ***x*** if $\boxed{y = \frac{k}{x}}$, where $k$ is a positive constant called the constant of variation.

In general, ***y*** **varies inversely with a power of** ***x*** if $\boxed{y = \frac{k}{x^n}}$, where $k$ and $n$ are positive numbers.

Note: In any example of inverse variation, as the independent variable increases through positive values, the dependent variable decreases. Thus, inverse variation is an example of a decreasing function.



Note: " Vary inversely" means exactly the same thing as " inversely proportional". The two phrases are interchangeable.

Example: The weight $w$ of an object varies inversely with the square of its distance $d$ from the center of the earth.

$w = \frac{k}{d^2}$

The amount of force $F$ (in pounds) needed to lift a heavy object with the help of a lever is inversely proportional to the length $l$ of the lever.

$F = \frac{k}{l}$

**Exercise #2** The intensity of electromagnetic radiation, such as light or radio waves, varies inversely with the square of the distance from its source. Radio station KPCC broadcasts a signal that is measured at 0.016 watts per square meter by a receiver one kilometer away.

a) Write a formula that gives signal strength as a function of distance.

let $i$ = intensity

$d$ = distance from source

$$\left.\begin{array}{l} \text{then } i = \dfrac{k}{d^2} \\ \text{if } i = 0.016\ \dfrac{w}{m^2},\ \text{then } d = 1\,km \end{array}\right\} \Rightarrow 0.016\ \frac{w}{m^2} = \frac{k}{1\,km^2} \Rightarrow k = 0.016\ \frac{w \cdot km^2}{m^2}$$

So $i = \dfrac{0.016}{d^2}$

b) If you live five kilometers from the station, what is the strength of the signal you will receive?

$$i = \frac{0.016}{d^2}$$

$$\text{if } d = 5\,km,\ \text{then } i = \frac{0.016\ \frac{w\,\cancel{km^2}}{m^2}}{25\ \cancel{km^2}} = 0.00064\ \frac{w}{m^2}$$

**More exercises**

1) For each function described below, (a) use the values in the table to find the constant of variation, *k*, and write *y* as a function of *x*; (b) fill in the rest of the table with the correct values.

I ) *y varies directly with x*

| *x* | *y* |
|---|---|
| 2 | |
| 5 | 1.5 |
| | 2.4 |
| 12 | |
| | 4.5 |

*II) y varies inversely with the square of x*

| *x* | *y* |
|---|---|
| 4 | |
| | 15 |
| 20 | 6 |
| 30 | |
| | 3 |

(A: I) y=0.3x; II) y=~~120~~ 2400/x)

2) The interest on an investment varies directly as the rate of interest. If the interest is \$48 when the interest rate is 5%, find the interest when the rate is 4.2%. (A: \$40.32)

3) Hooke's law for an elastic spring states that the distance a spring stretches varies directly with the force applied. If a force of 75 lb stretches a certain spring 16 inches, how much will a force of 200 lb stretch the spring? (A: 42 2/3 in)

4) If the temperature is constant, the pressure of a gas in a container varies inversely as the volume of the container. If the pressure is 10 lb per sq ft in a container with volume 3 c ft, what is the pressure in a container with volume 1.5 c ft? (A: 20)