ANSWERS

SELECTED PROBLEMS

SECTION 3.3

(18) (a) forward: $0 \le t < 1$ and $5 < t < 7$

backward: $1 < t < 5$

speeds up: $1 < t < 2$ and $5 < t < 6$

slows down: $0 \le t < 1$, $3 < t < 5$, $6 < t < 7$

(b) positive: $3 < t < 6$, negative: $0 \le t < 2$ and $6 < t < 7$

zero: $2 < t < 3$ and $7 < t < 9$

(c) $t = 0$ and $2 \le t \le 3$

(d) $7 \le t \le 9$

(26) $Q(t) = 200(30 - t)^2 \Rightarrow Q'(t) = 200(-60 + 2t)$

$Q'(10) = -8000$ gal/min is the rate the water is running at the end of 10 min.

$\frac{Q(10) - Q(0)}{10} = -10{,}000$ gal/min is the average rate the water flows during the first 10 min.

Note that the water is leaving the tank.

(28) (a) $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 \Rightarrow \left.\frac{dV}{dr}\right|_{r=2} = 16\pi\ ft^3/ft$

(b) When $r = 2$, $\frac{dV}{dr} = 16\pi \frac{ft^3}{ft}$ so that r changes by 1 unit, we expect V to change by approx 16π. Therefore, when r changes by 0.2 units V changes by approx $(16\pi)(0.2) = 3.2\pi \simeq 10.05\ ft^3$

Note that $V(2.2) - V(2) \simeq 11.09\ ft^3$

| section 3.9 |

(5) (a) $\frac{dV}{dt}$ volt/sec (b) $\frac{dI}{dt} = -\frac{1}{3}$ amp/sec

(c) $\frac{dV}{dt} = R\left(\frac{dI}{dt}\right) + I\left(\frac{dR}{dt}\right) \Rightarrow \frac{dR}{dt} = \frac{1}{I}\left(\frac{dV}{dt} - \frac{V}{I}\frac{dI}{dt}\right)$

(d) $\frac{dR}{dt} = \frac{1}{2}\left(1 - \frac{12}{2}\left(-\frac{1}{3}\right)\right] = \left(\frac{1}{2}\right)(3) = \frac{3}{2}$ ohms/sec

R is increasing

(10) $A = \pi r^2$, $\frac{dr}{dt} = 0.01$ cm/sec, $r = 50$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow \frac{dA}{dt}\Big|_{r=50} = \pi$ cm²/min.

(13) $\frac{dx}{dt} = 5$ ft/sec $x = 12, y = 5$

(a) $x^2 + y^2 = 169 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -12$ ft/sec

the ladder is sliding down the wall

(b) The area of the triangle formed by the ladder and walls is $A = \frac{1}{2}xy \Rightarrow \frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$

The area is changing at $\frac{1}{2}\left[12(-12) + 5(5)\right] = \frac{-119}{2}$
$= -59.5$ ft²/sec

(c) $\cos\theta = \frac{x}{13} \Rightarrow -\sin\theta\frac{d\theta}{dt} = \frac{1}{13}\cdot\frac{dx}{dt} \Rightarrow$

$\frac{d\theta}{dt} = -\frac{1}{13\sin\theta}\frac{dx}{dt} = -\frac{1}{5}(5) = -1$ rad/sec

(30) When s = length of the shadow and x = distance of the man from the streetlight, then $s = \frac{3}{5}x$.

(a) If l represents the tip of the shadow from the streetlight, then $l = s + x \Rightarrow \frac{dl}{dt} = \frac{ds}{dt} + \frac{dx}{dt}$ (velocity)

$\Rightarrow \left|\frac{dl}{dt}\right| = \left|\frac{3}{5}\frac{dx}{dt} + \frac{dx}{dt}\right| = 8$ ft/sec

(the speed ~~of~~ the tip of the shadow is moving along the ground)

(b) $\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dt} = \frac{3}{5}(-5) = -3$ ft/sec

(32) $\frac{d\theta}{dt} = -2$ rad/sec

$\frac{d\theta}{dt} = 1$ rad/sec