Section 4.6 – Applied Optimization Problems

How to Find the Absolute Extrema of a Continuous Function *f* on a Finite Closed Interval (4.1)
The Closed Interval Method

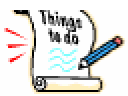
To find the absolute minimum and maximum values of a continuous function *f* on a closed interval [a,b]:

1. Find the critical numbers of *f*.
2. Find the values of *f* at the critical numbers and at the endpoints of the interval.
3. The largest of the values is the absolute maximum value; the smallest of the values is the absolute minimum value.

The Second Derivative Test for Local Extrema (4.4 Theorem 5)

Suppose f'' is continuous on an open interval that contains $x = c$.

1. If $f'(c) = 0$ and $f''(c) < 0$, then *f* has a local maximum at $x = c$.
2. If $f'(c) = 0$ and $f''(c) > 0$, then *f* has a local minimum at $x = c$.
3. If $f'(c) = 0$ and $f''(c) = 0$, then the test fails. The function *f* may have a local maximum, a local minimum, or neither.



Study Examples 1, 2, 3, and 5 from the textbook.

Problem 1 (#4) A rectangle has its base on the *x*-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

Problem 2 (#7) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single – strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

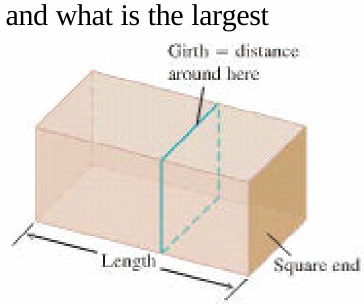
Problem 3 (#9) Your iron works has contracted to design and build a 500 cubic feet, square-based, open-top, rectangular steel holding tank for a paper company. The tank is to be made by welding thin stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible.

Problem 4 (#13) Two sides of a triangle have lengths a and b, and the angle between them is q. What value of q will maximize the triangle's area?

Problem 5 (#14) What are the dimensions of the lightest open-top right circular cylindrical can that will hold a volume of 1000 cubic centimeters?

Problem 6 (#18) A rectangle is to be inscribed under the arch of the curve $y = 4\cos(0.5x)$ from $x = -p$ to $x = p$. What are the dimensions of the rectangle with largest area, and what is the largest area?

Problem 7 (#20) The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 in. What distance will give a box with a square end the largest volume?

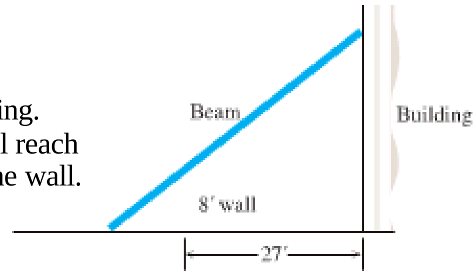


Problem 8 (#37) The height of an object moving vertically is given by

$s = -16t^2 + 96t + 112$, with s in feet and t in seconds. Find

a) the object's initial velocity
b) its maximum height and when it occurs
c) its velocity when $s = 0$.

Problem 9 (# 39) The 8-ft wall shown here stands 27 ft from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.

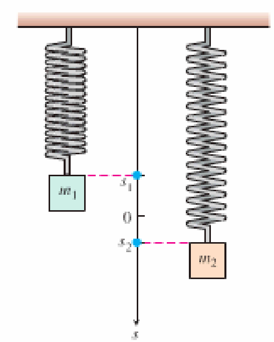


Problem 10 (# 40) The position of two particles on the *s*-axis are $s_1 = \sin t$ and $s_2 = \sin\left(t + \frac{p}{3}\right)$, with s_1 and s_2 in meters and *t* in seconds.

a) At what time(s) in the interval $[0, 2p]$ do the particles meet?
b) What is the farthest apart that the particles ever get?
c) When in the interval $[0, 2p]$ is the distance between the particles changing the fastest?

Problem 11 (# 46) Two masses hanging side by side from springs have positions $s_1 = 2\sin t$ and $s_2 = \sin 2t$, respectively.

a) At what times in the interval $0 < t$ do the masses pass each other?
b) When in the interval $0 \le t \le 2p$ is the vertical distance between the masses the greatest? What is this distance?



Problem 12 (#51) It costs you *c* dollars each to manufacture and distribute backpacks. If the backpacks sell at *x* dollars each, the number sold is given by

$$n = \frac{a}{x - c} + b(100 - x),$$

where *a* and *b* are positive constants. What selling price will bring a maximum profit?

Problem 13 (# 52) You operate a tour service that offers the following rates:$200 per person if 50 people (the minimum number to book the tour) go on the tour. For each additional person, up to a maximum of 80 people total, the rate per person is reduced $2. It costs $6000 (a fixed cost) plus $32 per person to conduct the tour. How many people dies it take to maximize your profit?

Problem 14 (#56) Suppose that $c(x) = x^3 - 20x^2 + 20{,}000x$ is the cost of manufacturing *x* items. Find a production level that will minimize the average cost of making *x* items.

Problem 15 (#33) Determine the dimensions of the rectangle of largest area that can be inscribed in a right triangle of dimensions 3, 4, and 5.