

VERTICAL SHIFTING (TRANSLATION)

Example #1

Use the graph of

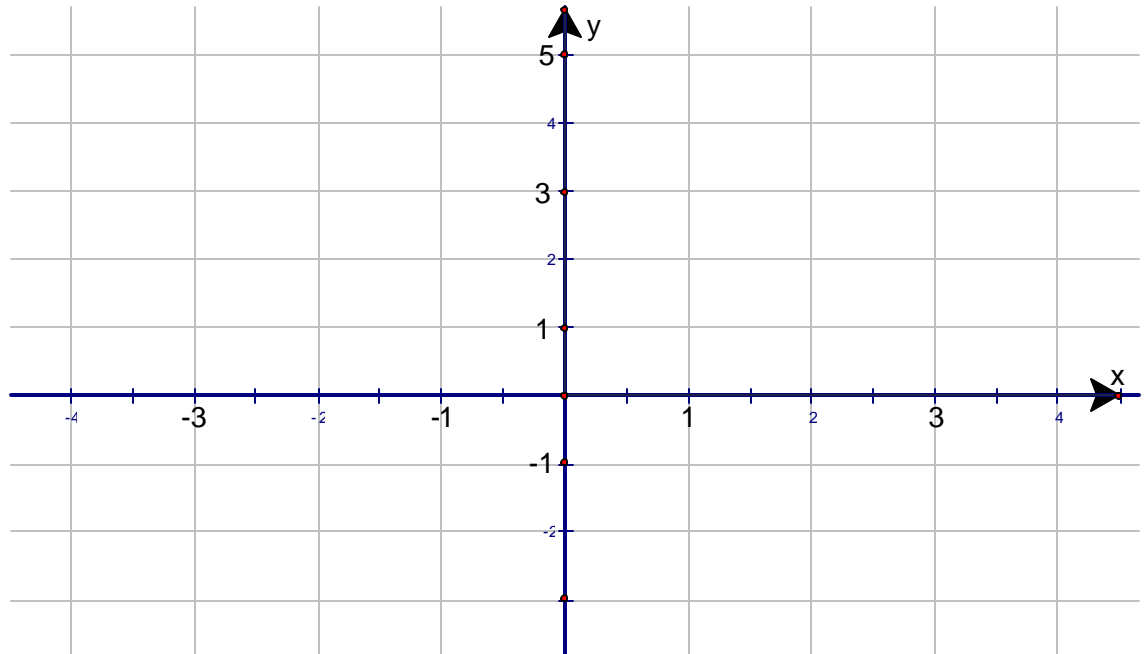
$$f(x) = x^2$$

to obtain the graphs of

$$g(x) = x^2 + 1$$

and

$$h(x) = x^2 - 2.$$



x	$f(x) = x^2$	$g(x) = x^2 + 1$	$h(x) = x^2 - 2$
-2			
-1			
0			
1			
2			

VERTICAL SHIFTING : A vertical shifting does not change the shape of the graph but simply translates it to another position in the plane.

Equation	How to obtain the graph	Example
$y = f(x) + k$ $k > 0$	Shift graph of $y = f(x)$ upward k units.	$g(x) = x^2 + 1$
$y = f(x) - k$ $k > 0$	Shift graph of $y = f(x)$ downward k units.	$h(x) = x^2 - 2$

HORIZONTAL SHIFTING (TRANSLATION)

Example #2

Use the graph of

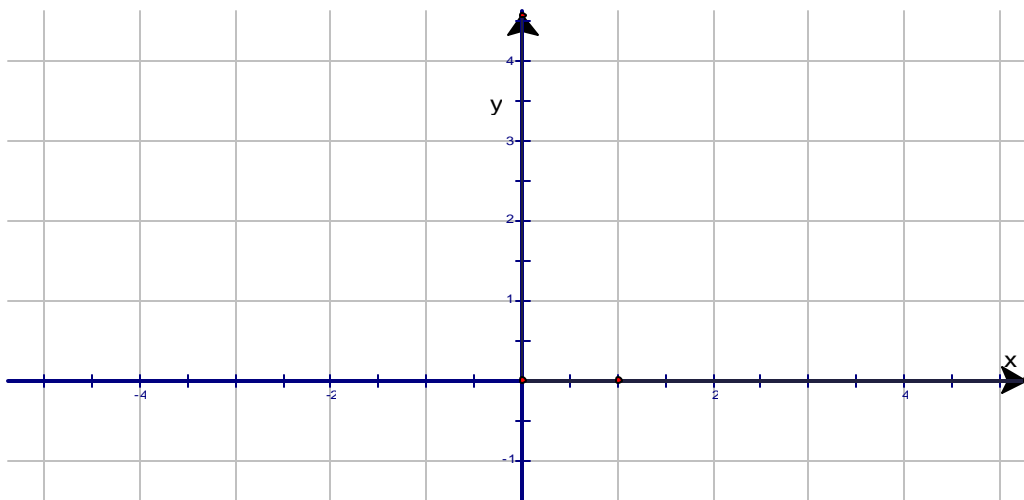
$$f(x) = x^2$$

to obtain the graphs of

$$g(x) = (x-1)^2$$

and

$$h(x) = (x+1)^2.$$



x	$f(x) = x^2$	$g(x) = (x-1)^2$	$h(x) = (x+1)^2$
-2			
-1			
0			
1			
2			

HORIZONTAL SHIFTING : A horizontal shifting doesn't change the shape of the graph but simply translates it to another position in the plane.

Equation	How to obtain the graph	Example
$y = f(x-h)$ $h > 0$	Shift graph of $y = f(x)$ to the right h units.	$g(x) = (x-1)^2$
$y = f(x+h)$ $h > 0$	Shift graph of $y = f(x)$ to the left h units.	$h(x) = (x+1)^2$

VERTICAL STRETCH AND COMPRESSION

Example #3

Use the graph of

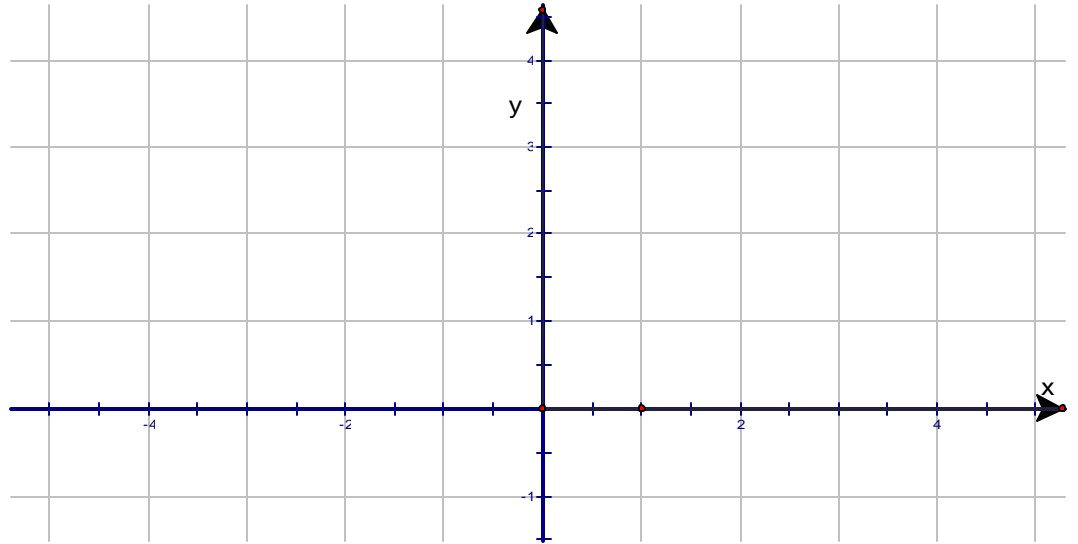
$$f(x) = |x|$$

to obtain the graphs of

$$g(x) = 2|x|$$

and

$$h(x) = \frac{1}{2}|x|$$



x	$f(x) = x $	$g(x) = 2 x $	$h(x) = \frac{1}{2} x $
-2			
-1			
0			
1			
2			

VERTICAL STRETCH AND COMPRESSION

Equation	How to obtain the graph	Example
$y = af(x)$ $a > 1$	Stretch the graph of $y = f(x)$ vertically by a factor of a .	$g(x) = 2 x $
$y = af(x)$ $0 < a < 1$	Compress the graph of $y = f(x)$ vertically by a factor of $\frac{1}{a}$.	$h(x) = \frac{1}{2} x $

HORIZONTAL COMPRESSION AND STRETCH

Example #4

Use the graph of

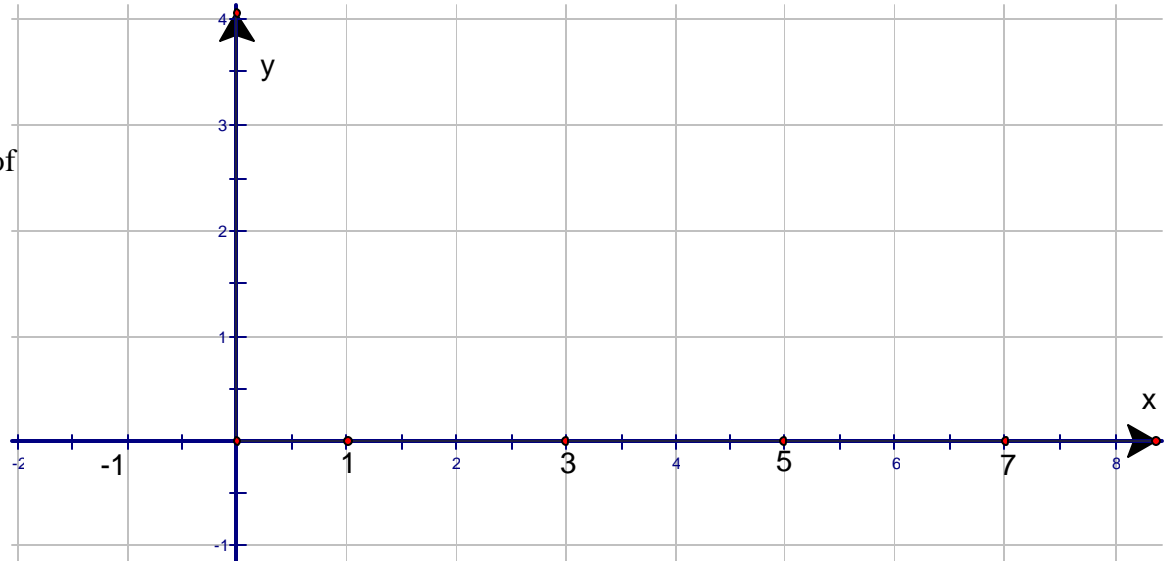
$$f(x) = \sqrt{x}$$

to obtain the graphs of

$$g(x) = \sqrt{2x}$$

and

$$h(x) = \sqrt{\frac{1}{2}x}.$$



x	$f(x) = \sqrt{x}$	$g(x) = \sqrt{2x}$	$h(x) = \sqrt{\frac{1}{2}x}$
0			
1			
4			
9			

Equation	How to obtain the graph	Example
$y = f(ax)$ $a > 1$	Compress the graph of $y = f(x)$ horizontally by a factor of a .	$g(x) = \sqrt{2x}$
$y = f(ax)$ $0 < a < 1$	Stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$.	$h(x) = \sqrt{\frac{1}{2}x}$

REFLECTION ABOUT THE AXES

Example #5

Use the graph of

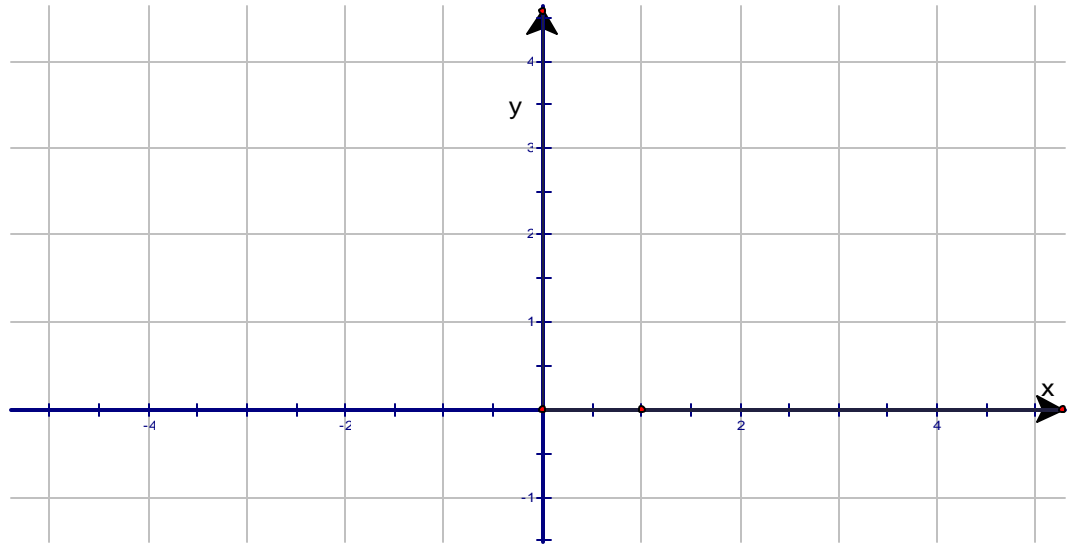
$$f(x) = \sqrt{x}$$

to obtain the graphs of

$$g(x) = -\sqrt{x}$$

and

$$h(x) = \sqrt{-x}$$



x	$f(x) = \sqrt{x}$	$g(x) = -\sqrt{x}$	$h(x) = \sqrt{-x}$
-4			
-1			
0			
1			
4			

REFLECTION ABOUT THE AXES

Equation	How to obtain the graph	Example
$y = -f(x)$	Reflect the graph of $y = f(x)$ about the x -axis.	$g(x) = -\sqrt{x}$
$y = f(-x)$	Reflect the graph of $y = f(x)$ about the y -axis.	$h(x) = \sqrt{-x}$

Exercise #1

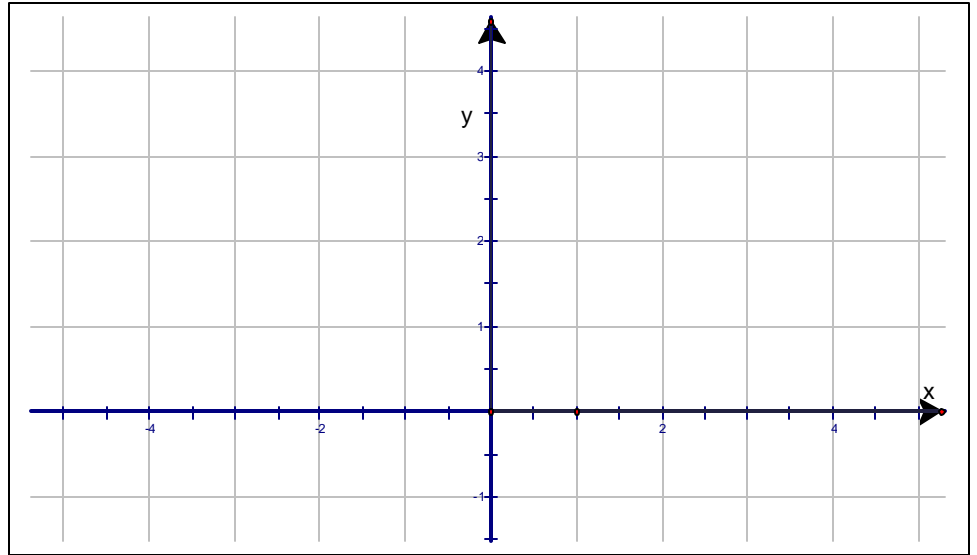
a) Use the graph of

$$f(x) = |x|$$

to obtain the graph of

$$g(x) = |x| + 2.$$

b) Find the domain and range.

**Exercise #2**

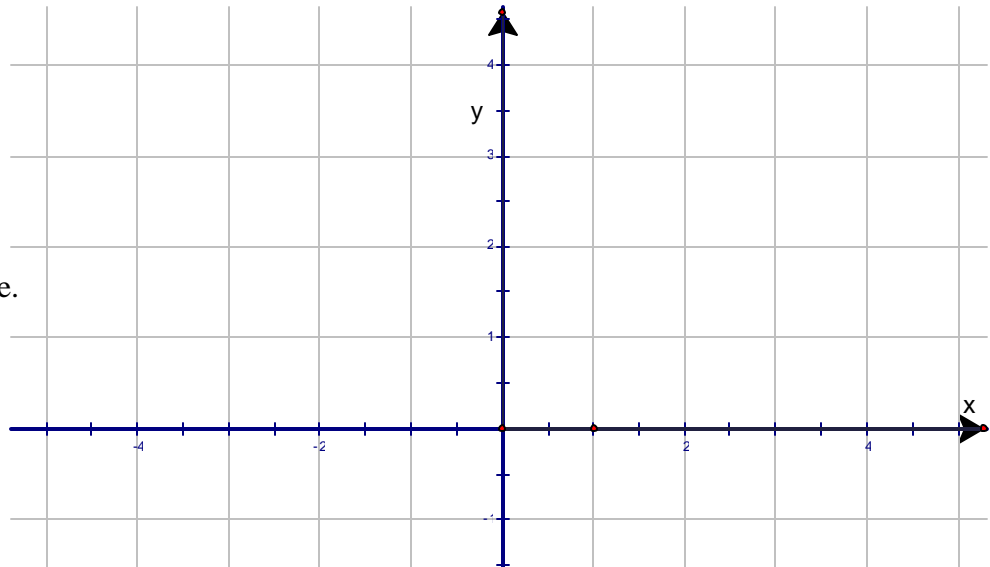
a) Use the graph of

$$f(x) = \sqrt{x}$$

to obtain the graph of

$$g(x) = \sqrt{x-3}.$$

b) Find the domain and range.

**Exercise #3**

Find the function that is finally graphed after the following transformations are applied to the graph of

a) $f(x) = \sqrt{x}$;

b) $g(x) = x^3$.

1) Shift left 3 units

2) Shift up 1 unit.

Exercise #4

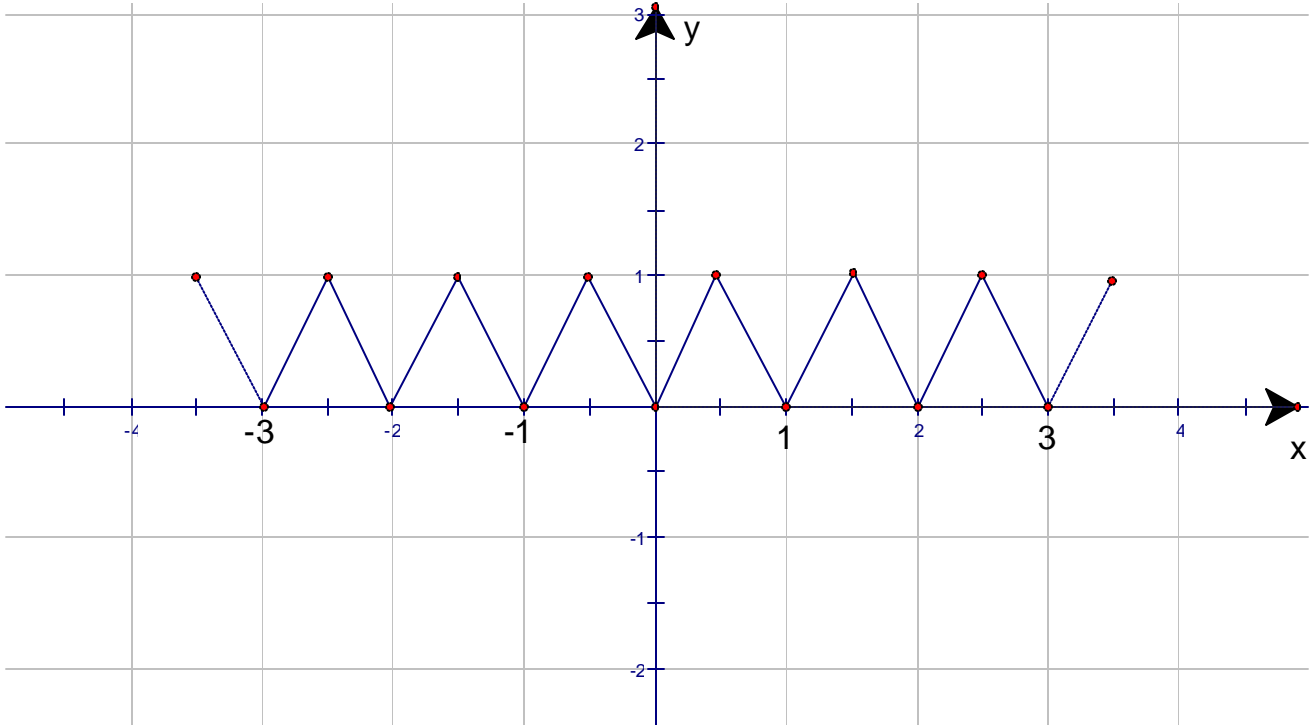
The graph of $y = f(x)$ is shown. Sketch the graph of each function:

a) $y = f(x+1)$

b) $y = f(x) - 2$.

c) $y = -f(x)$

d) $y = f(-x)$

**Exercise #5**

(2.7 #15)

Suppose the point $(8, 12)$ is on the graph of $y = f(x)$. Find a point on the graph of each function.

a) $y = f(x+4)$

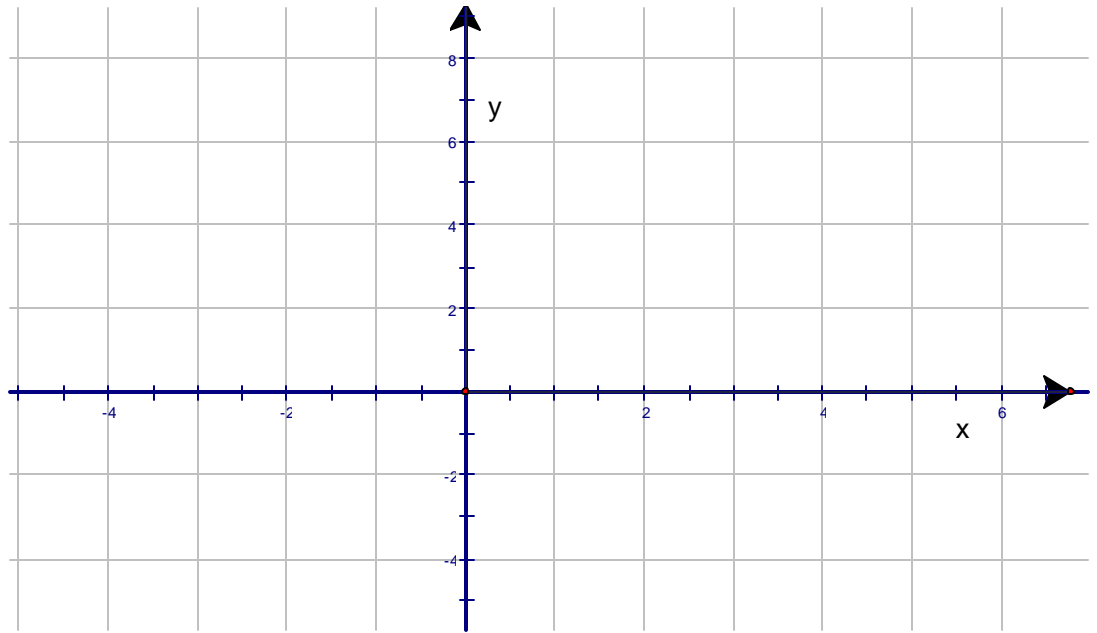
c) $y = \frac{1}{4}f(x)$

b) $y = f(x) + 4$

d) $y = 4f(x)$

Exercise #6 Graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages.
(2.7 #46, 51)

a) $f(x) = 2(x-2)^2 - 4$



b) $g(x) = |x+3| + 2$

