REVIEW
Sections 1.4, 1.6, and 1.7
Solving Equations and Inequalities

Definition The standard form of a quadratic or second degree equation in one variable is $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{R}; a \neq 0$.

Solving quadratic equations

(1) THE FACTORING METHOD – used to solve equations of the form $ax^2 + bx + c = 0$ that are factorable (see factoring methods on page 2)

Zero-Factor Property: The product of two factors equals zero if and only if one of the factors (or both) is zero.

$$AB = 0 \Leftrightarrow A = 0 \text{ or } B = 0$$

(2) EXTRACTION OF ROOTS – used to solve equations of the form

$x^2 = k$ or $(x - p)^2 = k$.

$\sqrt{x^2} = \sqrt{k}$ $\sqrt{(x-p)^2} = \sqrt{k}$

$x = \pm\sqrt{k}$ $x - p = \pm\sqrt{k}$

$x = p \pm \sqrt{k}$

(3) COMPLETING THE SQUARE $ax^2 + bx + c = 0$

Step 1: Coefficient of x^2 equal to 1.

Step 2: Constant isolated.

Step 3: Complete the square by adding $\left(\frac{1}{2} \cdot \text{coefficient of } x\right)^2$ to both sides of the equation and solve by the extraction of roots method.

(4) QUADRATIC FORMULA If $ax^2 + bx + c = 0$, then the solutions are given by:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition The discriminant of a quadratic equation is $\Delta = b^2 - 4ac$

Properties **(1)** If $a, b, c \in \mathbb{R}$, then:

If $\Delta > 0$, the equation has two distinct real solutions.

If $\Delta = 0$, the equation has one real (rational) solution.

If $\Delta < 0$, the equation has two complex (nonreal) solutions.

(2) If $a, b, c \in \mathbb{Q}$, then:

If Δ is a perfect square, the equation has **rational solutions**.

If Δ is not a perfect square, then the equation has **irrational solutions.**

Factoring a polynomial

1. GCF Factor out the greatest common factor (if any).

2. Special products

Two terms

$a^2 - b^2 = (a-b)(a+b)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Three terms

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

3. Factoring technique to factor out a trinomial $ax^2 + bx + c$

$a = 1$

$x^2 + bx + c = (x + \square)(x + \square)$

product = c
sum = b

$a \neq 1$

split the middle term bx

$ax^2 + bx + c = ax^2 + \square x + \square x + c$

product = ac
sum = b
then factor by grouping

4. If more than four term, factor by grouping.

Exercise #1 Solve by factoring (the zero-factor property):

a) $x^2 + 2x - 8 = 0$

b) $5x^2 - 3x = 2$

c) $-6x^2 + 7x = -10$

Exercise #2 Solve $(\text{in}\,\mathbb{C})$ by extracting roots (the square root property). Give exact answers.

a) $\frac{2x^2}{3} = 4$

b) $\left(t - \frac{1}{2}\right)^2 = \frac{3}{4}$

c) $1 - 3(x - 1)^2 = 28$

d) $(-2x + 5)^2 = -8$

Exercise #3 Solve $(\text{in}\,\mathbb{C})$ by completing the square. Give exact answers.

a) $x^2 - \frac{5}{3}x = 1$

b) $2p^2 + p - 10 = 0$

c) $-4x^2 + 8x = 7$

d) $x^2 + \sqrt{3}x - \frac{1}{4} = 0$

Exercise #4 Solve $(\text{in}\,\mathbb{C})$ by the quadratic formula. Give exact answers.

a) $x^2 - \frac{x}{2} + 1 = 0$

b) $\frac{1}{2}a^2 - 3 = -\frac{1}{4}a$

c) $3 - \frac{4}{x} - \frac{2}{x^2} = 0$

Exercise #5 Solve the following equations in $\mathbb{C}$.

a) $x^3 + 27 = 0$

b) $2x^7 - 128x = 0$

Exercise #6 a) Write (in standard form) a quadratic equation with rational coefficients that has $2 + \sqrt{3}$ as a solution.

b)Write (in standard form) a quadratic equation with integer coefficients that has 2 and $-\frac{1}{2}$ as solutions.

c) Write (in standard form) a quadratic equation with real coefficients that has 1+i as a solution.

Exercise #7 a) Determine k such that the solutions of $3x^2 + 4x = k$ are nonreal complex numbers.

b) Find the value(s) of k that will make the solutions of the following equation equal:

$(k-1)x^2 + (k-1)x + 1 = 0$

Exercise #8 Solve each equation for the indicated variable:

a) $3x^2 + xy + y^2 = 2$, for y;

b) $A = 2w^2 + 4lw$, for w;

c) $a^2 + b^2 = c^2$, for b

Exercise #9 Solve the following equations:

a) $2x^4 - 7x^2 + 5 = 0$

b) $(x+5)^{\frac{4}{3}} + (x+5)^{\frac{2}{3}} - 20 = 0$

c) $7x^{-2} - 10x^{-1} - 8 = 0$

Exercise #10 Solve the following equations:

a) $\frac{4x+3}{4} - \frac{2x}{x+1} = x$

b) $\frac{x}{x-3} = \frac{3}{x-3} + 3$

c) $\frac{4}{x^2+x-6} - \frac{1}{x^2-4} = \frac{2}{x^2+5x+6}$

d) $\frac{2x-5}{x} = \frac{x-2}{3}$

Exercise #11 Solve the following equations:

a) $\sqrt{4x+5} - 2 = 2x - 7$

b) $\sqrt{x} - \sqrt{x-12} = 2$

c) $(2x+5)^{\frac{1}{3}} - (6x-1)^{\frac{1}{3}} = 0$

Exercise #12 Solve each inequality. Write each solution set in interval notation and graph it.

a) $\frac{2x-5}{-8} \le 1 - x$

b) $-3 \le \frac{x-4}{-5} < 4$

c) $x^2 \le 9$

d) $x^2 + 4x > -1$

e) $x^3 + 4x^2 - 9x - 36 \ge 0$

f) $(x-5)^2(x+1) < 0$

Exercise #13 Solve each rational inequality. Write each solution set in interval notation.

a) $\frac{x+1}{x-4} > 0$

b) $\frac{x+3}{x-5} \le 1$

Answers: #1a) -4,2; b) -2/5, 1; c)-5/6, 2. #2 a) $\pm\sqrt{6}$; b) $\frac{1}{2} \pm \frac{\sqrt{3}}{2}$; c) $1 \pm 3i$; d) $\frac{5}{2} \pm \sqrt{2}i$. #3 a) $\frac{5 \pm \sqrt{61}}{6}$ b) -5/2,2; c) $\frac{2 \pm \sqrt{3}i}{2}$; d) $\frac{-\sqrt{3} \pm 2}{2}$. #4 a) $\frac{1 \pm \sqrt{15}i}{4}$; b) $\frac{-1 \pm \sqrt{97}}{4}$; c) $\frac{2 \pm \sqrt{10}}{3}$. #5 a) -3, $\frac{3 \pm 3\sqrt{3}i}{2}$; b) 0, $\pm 2, 1 \pm \sqrt{3}i, -1 \pm \sqrt{3}i$.#6 a) $x^2 - 4x + 1 = 0$; b) $2x^2 - 3x - 2 = 0$; c) $x^2 - 2x + 2 = 0$. #7) a) $k < -\frac{4}{3}$; b) 1,5 #8 a) $\frac{-x \pm \sqrt{8 - 11x^2}}{2}$; b) $\frac{-2l \pm \sqrt{4l^2 + 2A}}{2}$; c) $\pm\sqrt{c^2 - a^2}$. #9 a) $\pm 1, \pm\frac{\sqrt{10}}{2}$; b) -13, 3, $-5 \pm 5\sqrt{5}i$ c) -7/4,1/2. #10 a)3/5; b) $\varnothing$; c) -9; d) 3,5. #11 a) 5; b)16; c) 3/2. #12 a) $x \le \frac{1}{2}$; b) $x \in (-16, 19]$; c) $[-3, 3]$ d) $x < -2 - \sqrt{3}$ or $x > -2 + \sqrt{3}$; e) $x \in [-4, -3] \cup [3, \infty)$; f) $x < -1$. #13 a) $x < -1$ or $x > 4$; b) x<5.