

## REVIEW

### Sections 1.1, 1.4, and 1.6

### Solving Equations

Definition The standard form of a quadratic or second degree equation in one variable is  $ax^2 + bx + c = 0$  where  $a, b, c \in \mathbb{R}; a \neq 0$ .

#### Solving quadratic equations

**(1) THE FACTORING METHOD** – used to solve equations of the form

$ax^2 + bx + c = 0$  that are factorable (see factoring methods on page 2)

Zero-Factor Property: The product of two factors equals zero if and only if one of the factors (or both) is zero.

$$AB = 0 \Leftrightarrow A = 0 \text{ or } B = 0$$

**(2) EXTRACTION OF ROOTS** – used to solve equations of the form

$$\begin{array}{ll} x^2 = k & \text{or} & (x - p)^2 = k \\ \sqrt{x^2} = \sqrt{k} & & \sqrt{(x - p)^2} = \sqrt{k} \\ x = \pm\sqrt{k} & & x - p = \pm\sqrt{k} \\ & & x = p \pm \sqrt{k} \end{array}$$

**(3) COMPLETING THE SQUARE**  $ax^2 + bx + c = 0$

Step 1: Coefficient of  $x^2$  equal to 1.

Step 2: Constant isolated.

Step 3: Complete the square by adding  $\left(\frac{1}{2} \cdot \text{coefficient of } x\right)^2$  to both sides of the equation and solve by the extraction of roots method.

**(4) QUADRATIC FORMULA** If  $ax^2 + bx + c = 0$ , then the solutions are given by:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition The discriminant of a quadratic equation is  $\Delta = b^2 - 4ac$

Properties (1) If  $a, b, c \in \mathbb{R}$ , then:

- If  $\Delta > 0$ , the equation has two distinct real solutions.  
 If  $\Delta = 0$ , the equation has one real (rational) solution.  
 If  $\Delta < 0$ , the equation has two complex (nonreal) solutions.

(2) If  $a, b, c \in \mathbb{Q}$ , then:

- If  $\Delta$  is a perfect square, the equation has **rational solutions**.  
 If  $\Delta$  is not a perfect square, then the equation has **irrational solutions**.

### Factoring a polynomial

1. GCF Factor out the greatest common factor (if any).

2. Special products

Two terms

$$\begin{aligned} a^2 - b^2 &= (a - b)(a + b) \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \end{aligned}$$

Three terms

$$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)^2 \\ a^2 - 2ab + b^2 &= (a - b)^2 \end{aligned}$$

3. Factoring technique to factor out a trinomial  $ax^2 + bx + c$

$a = 1$

$$x^2 + bx + c = (x + \square)(x + \square)$$

product =  $c$   
sum =  $b$

$a \neq 1$

split the middle term  $bx$

$$ax^2 + bx + c = ax^2 + \square x + \square x + c$$

product =  $ac$   
sum =  $b$

then factor by grouping

4. If more than four term, factor by grouping.



- Exercise #8** a) Determine  $k$  such that the solutions of  $3x^2 + 4x = k$  are nonreal complex numbers.  
 b) Find the value(s) of  $k$  that will make the solutions of the following equation equal:  
 $(k-1)x^2 + (k-1)x + 1 = 0$

**Exercise #9** Solve each equation for the indicated variable:

- a)  $3x^2 + xy + y^2 = 2$ , for  $y$ ;  
 b)  $A = 2w^2 + 4lw$ , for  $w$ ; c)  $a^2 + b^2 = c^2$ , for  $b$

**Exercise #10** Solve the following equations:

- a)  $2x^4 - 7x^2 + 5 = 0$  d)  $\sqrt{y} + 9 = y + 3$   
 b)  $(x+5)^{\frac{4}{3}} + (x+5)^{\frac{2}{3}} - 20 = 0$  c)  $7x^{-2} - 10x^{-1} - 8 = 0$

**Exercise #11**

- a)  $x^2 + \sqrt{3}x - \frac{1}{4} = 0$  d)  $100 - 2(3x-1)^2 = 0$   
 b)  $3x(x+1) = 2x+2$  e)  $40 - 12m^2 = 0$   
 c)  $3x^2 \left(x + \frac{1}{2}\right) \left(2x - \frac{1}{3}\right) \left(5x - \frac{1}{2}\right) = 0$  f)  $5 - (x-1)^2 = (x-2)^2$

Answers:

#1 a)  $-6/11$ ; b)  $-2/7$ ; c)  $0$ .

#2a)  $-4, 2$ ; b)  $-2/5, 1$ ; c)  $-5/6, 2$ .

#3 a)  $\pm\sqrt{6}$ ; b)  $\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ ; c)  $1 \pm 3i$ ; d)  $\frac{5}{2} \pm \sqrt{2}i$ .

#4 a)  $\frac{5 \pm \sqrt{61}}{6}$ ; b)  $-5/2, 2$ ; c)  $\frac{2 \pm \sqrt{3}i}{2}$ .

#5 a)  $\frac{1 \pm \sqrt{15}i}{4}$ ; b)  $\frac{-1 \pm \sqrt{97}}{4}$ ; c)  $\frac{2 \pm \sqrt{10}}{3}$ ; d)  $\frac{-1 \pm \sqrt{41}}{4}$ .

#6 a)  $-3, \frac{3 \pm 3\sqrt{3}i}{2}$ ; b)  $0, \pm 2, 1 \pm \sqrt{3}i, -1 \pm \sqrt{3}i$ .

#7 a)  $x^2 - 4x + 1 = 0$ ; b)  $2x^2 - 3x - 2 = 0$ ; c)  $x^2 - 2x + 2 = 0$ .

#8 a)  $k < -\frac{4}{3}$ ; b)  $5$

#9 a)  $\frac{-x \pm \sqrt{8-11x^2}}{2}$ ; b)  $\frac{-2l \pm \sqrt{4l^2 + 2A}}{2}$ ; c)  $\pm\sqrt{c^2 - a^2}$ .

#10 a)  $\pm 1, \pm \frac{\sqrt{10}}{2}$ ; b)  $-13, 3, -5 \pm 5\sqrt{5}i$  c)  $-7/4, 1/2$ ; d)  $9$ .

#11 a)  $\frac{-\sqrt{3} \pm 2}{2}$ ; b)  $\frac{2}{3}, -1$ ; c)  $0, -\frac{1}{2}, \frac{1}{6}, \frac{1}{10}$ ; d)  $\frac{1 \pm 5\sqrt{2}}{3}$ ; e)  $\pm \frac{\sqrt{30}}{3}$ ; f)  $0, 3$ .