Math 71 – Fall 2011 Name:

QUIZ #3 @ 70 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Solve each equation

a) by completing the square (no credit will be given if using another method): $x^2+8x-5=0$.

b) by completing the square (no credit will be given if using another method): $2x^2+5x-3=0$.

c) by the method of your choice: $\left(x+\dfrac{2}{5}\right)^2=\dfrac{7}{25}$

d) by the method of your choice: $4x^2+49=0$

e) $C=\dfrac{kP_1P_2}{d^2}$ solve for d.

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2. Solve the following inequalities. Write the solution set using interval notation.

a) $x^2-2x-3\geq 0$ b) $(1-x)(x+3)(2x-3)>0$ c) $\dfrac{x+1}{x+3}<2$

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3. Graph each function on a separate coordinate system . Label all points. Then, state the domain, range, asymptote, and intercepts.

a) $f(x)=2^x$ b) $g(x)=\log_3 x$

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4. Let $f(x)=2x+3$ and $g(x)=\dfrac{1-3x}{x+2}$. Answer the following questions:

a) Find $(g\circ f)(x)$. b) $(f\circ g)(0)$ c) Find $f^{-1}(x)$. d) Find $g^{-1}(x)$

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5. Simplify the following expression: $\log_{10}\left(\log_3\left(\log_5 125\right)\right)$.

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6. Write each expression as a single logarithm whose coefficient is1.

a) $2\ln x^2-3\ln y+\dfrac{1}{2}\ln z$ b) $4\log_b x-2\log_b 6-\dfrac{1}{2}\log_b y$

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7. Expand each expression as much as possible. Where possible, evaluate without using a calculator.

a) $\log(1000x)$ b) $\ln\left(\dfrac{e^2}{5}\right)$ c) $\log_8\left(\dfrac{64}{\sqrt{x+1}}\right)$

MATH 71

Quiz 3 – Solutions

① ⓐ $x^2+8x-5=0$

- leading coeff. is 1.
- isolate the constant:

$$x^2+8x=5$$

- find missing term $\left(\frac{1}{2}\text{coef. } x\right)^2$

$$\left(\frac{1}{2}\cdot 8\right)^2=4^2=16$$

$$x^2+8x+16=5+16$$

$$(x+4)^2=21$$

$$\sqrt{(x+4)^2}=\sqrt{21}$$

$$x+4=\pm\sqrt{21}$$

$$\boxed{x=-4\pm\sqrt{21}}$$

ⓑ $2x^2+5x-3=0 \quad |\div 2$

- leading coeff. 1:

$$x^2+\frac{5}{2}x-\frac{3}{2}=0$$

- isolate the constant:

$$x^2+\frac{5}{2}x=\frac{3}{2}$$

- find missing term: $\left(\frac{1}{2}\text{coef. } x\right)^2$

$$\left(\frac{1}{2}\cdot\frac{5}{2}\right)^2=\left(\frac{5}{4}\right)^2=\frac{25}{16}$$

$$x^2+\frac{5}{2}x+\frac{25}{16}=\overset{8/}{\frac{3}{2}}+\frac{25}{16}$$

$$\left(x+\frac{5}{4}\right)^2=\frac{49}{16}$$

$$\sqrt{x+\frac{5}{4}}=\sqrt{\frac{49}{16}}$$

$$x+\frac{5}{4}=\pm\sqrt{\frac{49}{16}}$$

$$x=-\frac{5}{4}\pm\frac{7}{4} \quad \begin{cases} -\frac{5}{4}+\frac{7}{4}=\frac{1}{2} \\ -\frac{5}{4}-\frac{7}{4}=\frac{-12}{4}=-3 \end{cases}$$

$$\boxed{x\in\left\{\frac{1}{2},-3\right\}}$$

ⓒ $\left(x+\frac{2}{5}\right)^2=\frac{7}{25}$

The Square Root Method:

$$\sqrt{\left(x+\frac{2}{5}\right)^2}=\sqrt{\frac{7}{25}}$$

$$x+\frac{2}{5}=\pm\sqrt{\frac{7}{25}}$$

$$x=-\frac{2}{5}\pm\frac{\sqrt{7}}{5}$$

$$\boxed{x=\frac{-2\pm\sqrt{7}}{5}}$$

ⓓ $4x^2+49=0$

The Square Root Method:

$$4x^2=-49$$

$$x^2=\frac{-49}{4}$$

$$\sqrt{x^2}=\sqrt{\frac{-49}{4}}$$

$$x=\pm\sqrt{\frac{-49}{4}}=\pm\frac{7}{2}i$$

$$\boxed{x=\pm\frac{7}{2}i}$$

ⓔ $C=\frac{kP_1P_2}{d^2}$, find d

$$Cd^2=kP_1P_2$$

$$d^2=\frac{kP_1P_2}{C}$$

$$\sqrt{d^2}=\sqrt{\frac{kP_1P_2}{C}}$$

$$\boxed{d=\pm\sqrt{\frac{kP_1P_2}{C}}}$$

usually, only $d=\sqrt{\frac{kP_1P_2}{C}}$

② (a) $x^2-2x-3 \geq 0$

let $y = x^2-2x-3$

We need to find x such that $y \geq 0$

Note that $y = x^2-2x-3$ is a parabola opening up

with x-n given by

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3, \quad x=-1$

So $y \geq 0$ iff $x \leq -1$ OR $x \geq 3$

Therefore,

$\boxed{x \in (-\infty, -1] \cup [3, \infty)}$

(b) $(1-x)(x+3)(2x-3) > 0$

we'll study the sign of each factor, then the sign of the product.

x	$-\infty$ $\quad$ -3 $\quad$ 1 $\quad$ $\frac{3}{2}$ $\quad$ ∞
$1-x$	+ + + + 0 − − − −
$x+3$	− − − 0 + + + + +
$2x-3$	− − − − − − 0 + +
$(1-x)(x+3)(2x-3)$	+ 0 − 0 + 0 −

Therefore,

$(1-x)(x+3)(2x-3) > 0$ iff

$\boxed{x \in (-\infty, -3) \cup (1, \tfrac{3}{2})}$

(c) $\frac{x+1}{x+3} < 2$

$\frac{x+1}{x+3} - \frac{2}{1}^{x+3} < 0$

$\frac{x+1-2(x+3)}{x+3} < 0$

$\frac{x+1-2x-6}{x+3} < 0$

$\frac{-x-5}{x+3} < 0$

We'll study the sign of each factor, then the sign of the rational expression

x	$-\infty$ $\quad$ -5 $\quad$ -3 $\quad$ ∞
$-x-5$	+ + + 0 − − − −
$x+3$	− − − − − 0 + +
$\frac{-x-5}{x+3}$	− 0 + | −

Therefore, $\frac{-x-5}{x+3} < 0$

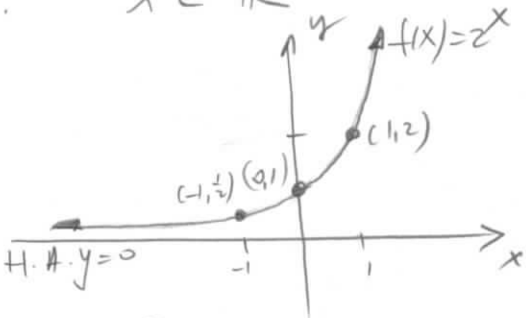
iff $\boxed{x \in (-\infty, -5) \cup (-3, \infty)}$

(3) (a) $f(x) = 2^x$

exponential fct.

Domain: $x \in \mathbb{R}$

x	y
-1	$\frac{1}{2}$
0	1
1	2



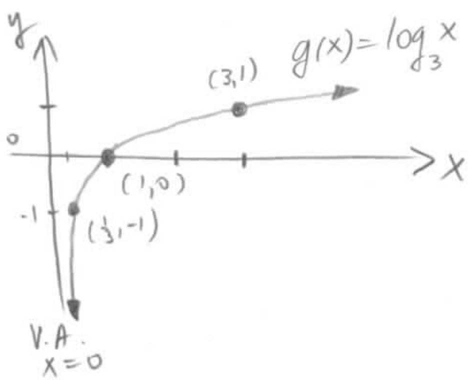
Domain: $x \in \mathbb{R}$
Range: $y > 0$
H.A. $y = 0$
x-n: none
y-n: $(0,1)$

(b) $g(x) = \log_3 x$

logarithmic fct.

Domain: $x > 0$

x	y
$\frac{1}{3}$	-1
1	0
3	1



(4) $f(x) = 2x+3$

$$g(x) = \frac{1-3x}{x+2}$$

(a) $(g \circ f)(x) = g(f(x))$

$$= g(2x+3)$$

$$= \frac{1-3(2x+3)}{(2x+3)+2}$$

$$(g \circ f)(x) = \frac{1-6x-9}{2x+3+2}$$

$$\boxed{(g \circ f)(x) = \frac{-6x-8}{2x+5}}$$

(b) $(f \circ g)(0) = f(g(0))$

$$= f\left(\tfrac{1}{2}\right)$$

$$= 2 \cdot \tfrac{1}{2} + 3 = 4$$

$$\boxed{(f \circ g)(0) = 4}$$

(c) $f(x) = 2x+3$

1st let $y = 2x+3$

2nd solve for x

$$2x = y-3$$

$$x = \frac{y-3}{2}$$

3rd $x \leftrightarrow y$

$$y = \frac{x-3}{2}$$

Therefore, $\boxed{f^{-1}(x) = \frac{x-3}{2}}$

(d) $g(x) = \dfrac{1-3x}{x+2}$

1st let $y = \dfrac{1-3x}{x+2}$

2nd solve for x

$$y(x+2) = 1-3x$$

$$yx + 2y = 1-3x$$

$$yx + 3x = 1-2y$$

$$x(y+3) = 1-2y$$

$$x = \frac{1-2y}{y+3}$$

3rd $x \leftrightarrow y$

$$y = \frac{1-2x}{x+3}$$

So, $\boxed{g^{-1}(x) = \frac{1-2x}{x+3}}$

(5) $\log_{10}(\log_3(\log_5 125)) =$

$\log_{10}(\log_3 3) =$

$\log_{10} 1 = \boxed{0}$

(6) (a) $2\ln x^2 - 3\ln y + \frac{1}{2}\ln z =$

$= \ln(x^2)^2 - \ln y^3 + \ln z^{\frac{1}{2}}$

$= \ln \frac{x^4}{y^3} + \ln\sqrt{z} = \boxed{\ln \frac{x^4\sqrt{z}}{y^3}}$

(b) $4\log_6 x - 2\log_5 6 - \frac{1}{2}\log_6 y =$

$= \log_5 x^4 - \log_6 6^2 - \log_5 y^{1/2}$

$= \log_5 x^4 - (\log_5 36 + \log_5 \sqrt{y})$

$= \log_6 x^4 - \log_5 36\sqrt{y}$

$= \boxed{\log_5 \frac{x^4}{36\sqrt{y}}}$

(7) (a) $\log(1000x) =$

$= \log 1000 + \log x$

$= \boxed{3 + \log x}$

(b) $\ln\left(\frac{e^2}{5}\right) = \ln e^2 - \ln 5$

$= \boxed{2 - \ln 5}$

(c) $\log_8 \frac{64}{\sqrt{x+1}} =$

$= \log_8 64 - \log_8 \sqrt{x+1}$

$= 2 - \log_8 (x+1)^{1/2}$

$= \boxed{2 - \frac{1}{2}\log_8(x+1)}$