TEST #2 @ 200 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

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1. Consider $f(x)=\dfrac{2x^2+x-3}{x^3+2x^2-9x-18}$. Graph the function showing all work. Label everything.

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2. Let $f(x)=2\ln(x+3)+1$.

a) Graph the function using transformations. Clearly show how you're obtaining the graph, that is, show all equations, their meaning, and the corresponding graphs.

b) State the domain, range, and asymptote.

c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse? Explain.

e) Find $f^{-1}(x)$.

f) State the domain, range, and asymptote for the inverse function $f^{-1}(x)$.

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3. Solve the following equations. Give exact answers as well as approximations (when appropriate). Write conditions (if any).

a) $\log(x+6)-\log(x+2)=\log x$

b) $3^{2x+1}=4^{3x-2}$

c) $\sqrt{\ln x}=\ln x$

d) $\ln x+\ln x^2=3$

e) Solve for t: $A=P\left(1+\dfrac{r}{n}\right)^{nt}$

f) $6\cdot 2^x=7\cdot 5^x$

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4. Graph the solution set of the following system of inequalities. Clearly show your work.

$$\begin{cases} y>3^x-1 \\ y\le \log_2(x+2) \end{cases}$$

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5. State whether each statement is TRUE or FALSE. No proofs needed.

a) $\log(AB)=\log A+\log B$

b) $\dfrac{\log a}{\log b}=\log a-b$

c) $\log\sqrt{x}=\dfrac{1}{2}\log x$

d) $\log(a+b)=\log a+\log b$

e) $\sqrt{\log x}=\log\left(x^{\frac{1}{2}}\right)$

f) $\log\left(\dfrac{a}{b}\right)=\dfrac{\log a}{\log b}$

g) $\log a=\ln a$

h) $p\log A=\log A^p$

6. Solve the following system using matrice

$$\begin{cases} 2x-3y-z=13 \\ -x+2y-5z=6 \\ 5x-y-z=49 \end{cases}$$

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7. Solve the following system using matrices:

$$\begin{cases} x+3y-2z-w=9 \\ 4x+y+z+2w=2 \\ -3x-y+z-w=-5 \\ x-y-3z-2w=2 \end{cases}$$

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8. Let $A=\begin{pmatrix} -3 & 1 & 1 \\ 1 & -1 & 2 \\ 5 & 0 & 0 \end{pmatrix}$ $B=\begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

Do the following operations. If not defined, say so and explain why.

a) $2A+B$ b) AB c) B^2

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9. The number of bacteria present in a culture after t hours is given by $N(t)=1200e^{0.57t}$.

a) What was the initial number of bacteria? Write your answer using function notation.

b) How many bacteria will there be after 3 hours? Write your answer using function notation.

c) How long will it be before there are 20,000 bacteria? Write your answer using function notation.

d) What is the doubling time?

e) Find the inverse function and explain its meaning.

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10. Find the inverse of each function:

a) $f(x)=\dfrac{3x+2}{4x-1}$

b) $g(x)=\log_2\left(\dfrac{x-1}{x+3}\right)$

c) $h(x)=4\sqrt[3]{x+5}$

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11. The number of vibrations per second (the pitch) of a steel guitar string, *n*, varies directly as the square root of the tension *T* and inversely as the length of the string, l. If the number of vibrations per second is 5 when the tension is 225 kg and the length is 0.60 m, find the number of vibrations per second when the tension is 196 kg and the length is 0.65m.

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12. a) If you need $30,000 eight years from now, what is the minimum amount of money you need to deposit into a bank account that pays 4% annual interest, compounded monthly?

b) If you deposit $5000 into a bank account that pays 4% interest compounded continuously, how long will it be until you have $30,000 ?

($A=Pe^{rt}$, $A=P\left(1+\dfrac{r}{n}\right)^{nt}$)

EXTRA CREDIT

#1 @ 5 points

Five different stories are given below. Following the stories are five formulas. Match each formula to the story it models and state what the variables represent. You should assume that the constants P_0, r, B, A are all positive.

a) The percent of a lake's surface covered by algae, initially at 35%, was halved each year since the passage of anti-pollution laws.

b) The amount of charge on a capacitor in an electric circuit decreases by 30% every second.

c) Polluted water is passed through a series of filters. Each filter removes all but 30% of the remaining impurities from the water.

d) In 1920, the population of a town was 3000 people. Over the course of the next 50 years, the town grew at a rate of 10% per decade.

e) In 1920, the population of a town was 3000 people. Over the course of the next 50 years, the town grew at a rate of 250 people per year.

(i) $f(x) = P_0 + rx$ (ii) $g(x) = P_0(1+r)^x$ (iii) $h(x) = B(0.7)^x$

(iv) $j(x) = B(0.3)^x$ (v) $k(x) = A(2)^{-x}$

#2 @ 3 points

Find the inverse function for $f(x) = \dfrac{5 \cdot e^x + 1}{3 \cdot e^x - 4}$.

#3 @ 4 points

A small fast-food chain has restaurants in Santa Monica, Long Beach, and Anaheim. Only hamburgers, hot dogs, and milk shakes are sold by this chain. On a certain day, sales were distributed according to the following matrix:

Number of items sold

SM =Santa Monica, LB =Long Beach, A = Anaheim

$$\begin{array}{l} \\ \text{Hamburgers} \\ \text{Hot dogs} \\ \text{Milk shakes} \end{array} \begin{array}{c} \begin{array}{ccc} \text{SM} & \text{LB} & \text{A} \end{array} \\ \begin{bmatrix} 4000 & 1000 & 3500 \\ 400 & 300 & 200 \\ 700 & 500 & 9000 \end{bmatrix} \end{array} = A$$

The price of each item is given by the following matrix:

$$\begin{array}{ccc} \text{Hambg.} & \text{Hot Dog} & \text{Milk Shake} \end{array}$$
$$\begin{bmatrix} \$0.90 & \$0.80 & \$1.10 \end{bmatrix}$$

a) Calculate the product BA.
b) Interpret the entries in the product matrix BA.

#4 @ 3 points

Find a matrix $A \neq 0$ such that $A^2 = 0$.

TEST #2 - SOLUTIONS

① $f(x) = \dfrac{2x^2+x-3}{x^3+2x^2-9x-18}$

$= \dfrac{(2x+3)(x-1)}{x^2(x+2)-9(x+2)} = \dfrac{(2x+3)(x-1)}{(x+2)(x^2-9)}$

$f(x) = \dfrac{(2x+3)(x-1)}{(x+2)(x-3)(x+3)}$

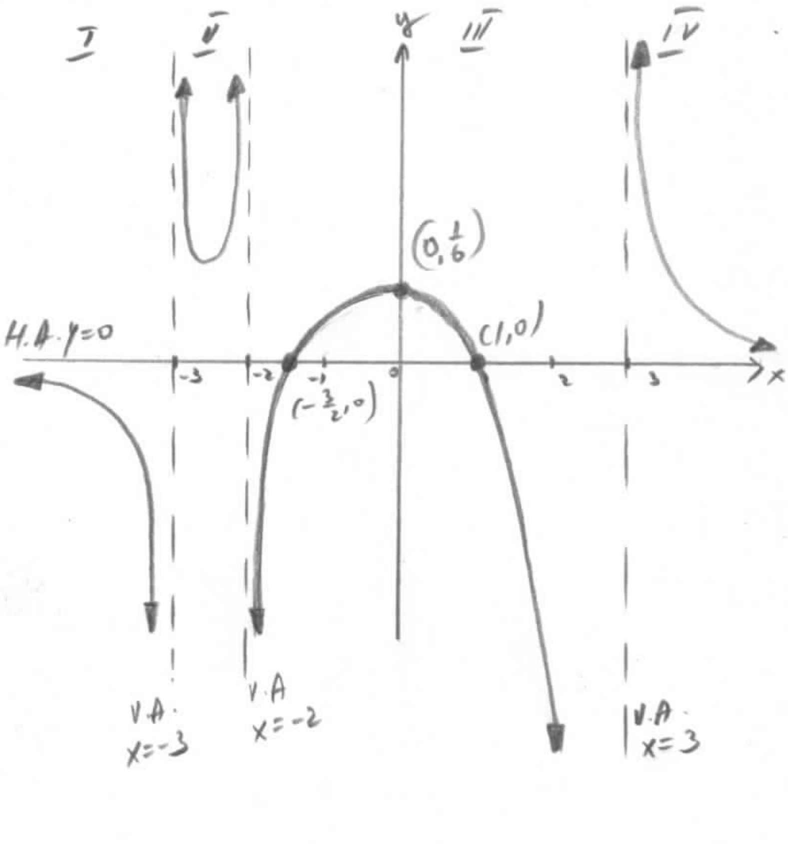
1. Domain: $x \in \mathbb{R} \setminus \{-2, 3, -3\}$

2. V.A. $x=-2$, $x=3$, $x=-3$

H.A. $y=0$

3. x-int: $y=0$ iff
$(2x+3)(x-1)=0$ iff
$x=-\frac{3}{2}$, $x=1$

4. y-int: if $x=0$, $y=\frac{-3}{-18}=\frac{1}{6}$



5. Test points:

I let $x=-5$, then $y=\dfrac{(-)(-)}{(-)(-)(-)}$ so $y<0$

II let $x=-2.5$, then

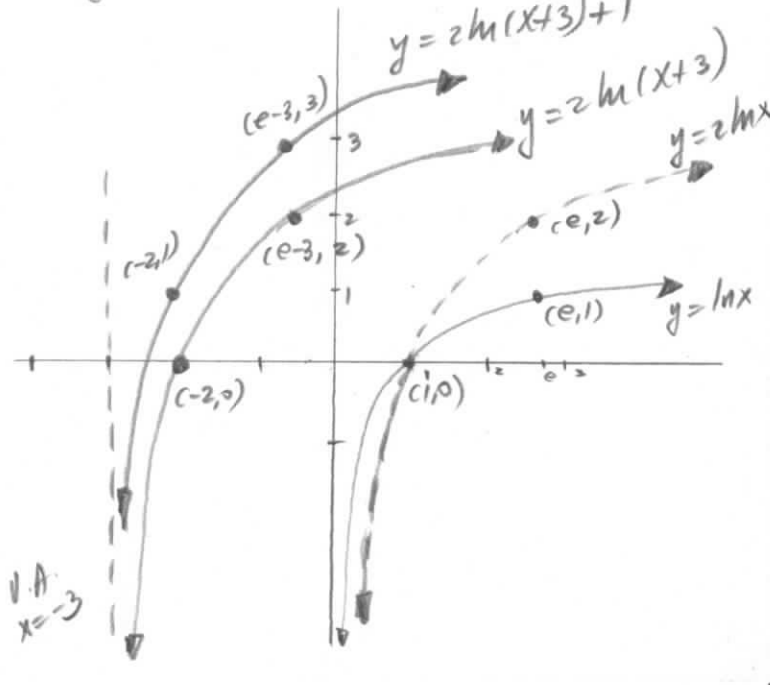
$y=\dfrac{(2(-2.5)+3)(-2.5-1)}{(-2.5+2)(-2.5-3)(-2.5+3)}$

$y=\dfrac{(-)(-)}{(-)(-)(+)} = (+)$ so $y>0$

III let $x=5$, then $y=\dfrac{(+)(+)}{(+)(+)(+)}$

② $f(x)=2\ln(x+3)+1$

(a) 1st $y=\ln x$
and $y=2\ln x$ vertical stretch by 2
3rd $y=2\ln(x+3)$ shift left 3 units
4th $y=2\ln(x+3)+1$ shift up 1 unit



(b) Domain: $x \in (-3, \infty)$
Range: $y \in \mathbb{R}$
V.A. $x = -3$

(c) y-int: let $x=0$, then
$y = 2\ln 3 + 1$

$\boxed{(0, 2\ln 3 + 1)}$

x-int let $y=0$, then
solve $2\ln(x+3)+1=0$

$$\ln(x+3) = \frac{-1}{2}$$

$$x+3 = e^{-\frac{1}{2}}$$

$$x = -3 + \frac{1}{\sqrt{e}}$$

$\boxed{\left(-3+\frac{1}{\sqrt{e}}, 0\right)}$

(d) Yes, the function is one-to-one, therefore it has an inverse.

(e) 1. let $y = 2\ln(x+3)+1$
2. Solve for x

$$y-1 = 2\ln(x+3)$$

$$\ln(x+3) = \frac{y-1}{2}$$

$$x+3 = e^{\frac{y-1}{2}}$$

$$x = -3 + e^{\frac{y-1}{2}}$$

3. $x \longleftrightarrow y$

$$y = -3 + e^{\frac{x-1}{2}}$$

so $\boxed{f^{-1}(x) = -3 + e^{\frac{x-1}{2}}}$

(f) Domain: $x \in \mathbb{R}$
Range: $y \in (-3, \infty)$
H.A. $y = -3$

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(3) (a) $\log(x+6) - \log(x+2) = \log x$

Conditions: $\begin{cases} x+6>0 \\ x+2>0 \\ x>0 \end{cases} \iff$

$\begin{cases} x>-6 \\ x>-2 \\ x>0 \end{cases} \iff x>0$

$\log \frac{x+6}{x+2} = \log x$
The logarithmic fct. is one-to-one $\Rightarrow$

$$\Rightarrow \frac{x+6}{x+2} = x$$

$$x+6 = x(x+2)$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$x=-3$ OR $x=2$
does not satisfy the condition

Therefore, $\boxed{x \in \{2\}}$

(b) $3^{2x+1} = 4^{3x-2}$ $\quad / \ln$

$\ln 3^{2x+1} = \ln 4^{3x-2}$

$(2x+1)\ln 3 = (3x-2)\ln 4$

$2x\ln 3 + \ln 3 = 3x\ln 4 - 2\ln 4$

$\ln 3 + 2\ln 4 = 3x\ln 4 - 2x\ln 3$

$\ln 3 + \ln 4^2 = x(3\ln 4 - 2\ln 3)$

$\ln(3\cdot 16) = x(\ln 4^3 - \ln 3^2)$

$$\boxed{x = \frac{\ln 48}{\ln\left(\frac{64}{9}\right)}}$$

$\boxed{x \approx 1.97}$

(c) $\sqrt{\ln x} = \ln x$

Conditions: $\begin{cases} \ln x \geq 0 \\ x > 0 \end{cases} \Leftrightarrow \begin{cases} x \geq 1 \\ x > 0 \end{cases}$

so $x \geq 1$

let $\ln x = t$

then, the equation becomes

$\sqrt{t} = t$ $\quad /^2$

$(\sqrt{t})^2 = t^2$

$t = t^2$

$t^2 - t = 0$

$t(t-1) = 0$ $\Big\langle$ $t = 0$ (check $\sqrt{0} = 0$) OR $t = 1$ (check $\sqrt{1} = 1$)

if $t = 0$, then

$\ln x = 0$

$x = e^0 = 1 \geq 1$

if $t = 1$, $\ln x = 1$

$x = e^1 = e \geq 1$

Therefore, $\boxed{x \in \{1, e\}}$

(d) $\ln x + \ln x^2 = 3$

Conditions: $\begin{cases} x > 0 \\ x^2 > 0 \end{cases} \Leftrightarrow \begin{cases} x > 0 \\ x \neq 0 \end{cases}$

so $x > 0$

$\ln x^3 = 3$

$e^3 = x^3$

The cubic fct. $y = x^3$ is one-to-one, therefore

$x = e > 0$

so $\boxed{x \in \{e\}}$

(e) $A = P\left(1 + \frac{r}{n}\right)^{nt}$, find t

$\left(1 + \frac{r}{n}\right)^{nt} = \frac{A}{P}$ $\quad / \ln$

$\ln\left(1 + \frac{r}{n}\right)^{nt} = \ln \frac{A}{P}$

$nt \ln\left(1 + \frac{r}{n}\right) = \ln \frac{A}{P}$

$$\boxed{t = \frac{\ln\left(\frac{A}{P}\right)}{n \ln\left(1 + \frac{r}{n}\right)}}$$

(f) $6\cdot 2^x = 7\cdot 5^x \quad / \ln$

$\ln(6\cdot 2^x) = \ln(7\cdot 5^x)$

$\ln 6 + \ln 2^x = \ln 7 + \ln 5^x$

$\ln 6 + x\ln 2 = \ln 7 + x\ln 5$

$x\ln 2 - x\ln 5 = \ln 7 - \ln 6$

$x(\ln 2 - \ln 5) = \ln 7 - \ln 6$

$$\boxed{x = \frac{\ln\left(\frac{7}{6}\right)}{\ln\left(\frac{2}{5}\right)} \approx -0.38}$$

(h) $\begin{cases} y > 3^x - 1 \\ y \le \log_2(x+2) \end{cases}$

1. $y > 3^x - 1$

Boundary curve: $y = 3^x - 1$ (dashed)

We'll graph $y = 3^x$, then shift down 1 unit.

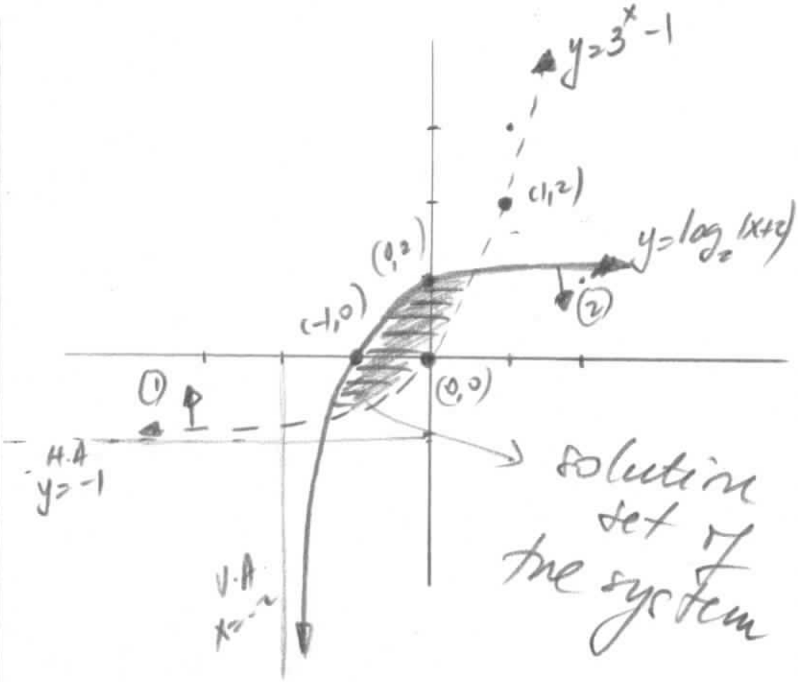
The sol. set is all points above the curve

2. $y \le \log_2(x+2)$

Boundary curve: $y = \log_2(x+2)$ (solid)

We'll graph $y = \log_2 x$, then shift left 2 units.

The sol. set is all points below the curve, including the curve.



(5) (a) TRUE (e) False

(b) False (f) False

(c) TRUE (g) False

(d) False (h) True

(6)
$$\left[\begin{array}{ccc|c} 2 & -3 & -1 & 13 \\ -1 & 2 & -5 & 6 \\ 5 & -1 & -1 & 49 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\left[\begin{array}{ccc|c} -1 & 2 & -5 & 6 \\ \boxed{2} & -3 & -1 & 13 \\ \boxed{5} & -1 & -1 & 49 \end{array}\right] \xrightarrow[R_3 \to 5R_1 + R_3]{R_2 \to 2R_1 + R_2}$$

$$\left[\begin{array}{ccc|c} -1 & 2 & -5 & 6 \\ 0 & 1 & -11 & 25 \\ 0 & \boxed{9} & -26 & 79 \end{array}\right] \xrightarrow{R_3 \to -9R_2 + R_3}$$

$$\left[\begin{array}{ccc|c} -1 & 2 & -5 & 6 \\ 0 & 1 & -11 & 25 \\ 0 & 0 & 73 & -146 \end{array}\right]$$

Using back substitution =>

$73z = -146 \Rightarrow z = -2$

$y - 11z = 25 \Rightarrow y - 11(-2) = 25$

$y + 22 = 25$

$y = 3$

$-x + 2y - 5z = 6 \Rightarrow$

$-x + 2(3) - 5(-2) = 6$

$-x + 6 + 10 = 6$

$x = 10$

The solution is $(10, 3, -2)$.

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(7)
$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ \boxed{4} & 1 & 1 & 2 & 2 \\ \boxed{-3} & -1 & 1 & -1 & -5 \\ \boxed{1} & -1 & -3 & -2 & 2 \end{array}\right]^{3} \xrightarrow[\substack{R_3 \to 3R_1 + R_3 \\ R_4 \to -R_1 + R_4}]{R_2 \to -4R_1 + R_2}$$

$$\begin{array}{c} \\ \\ \\ 2 \end{array}\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & \boxed{8} & -5 & -4 & 22 \\ 0 & -4 & -1 & -1 & -7 \end{array}\right] \xrightarrow{R_3 \to 2R_4 + R_3}$$

$$\begin{array}{c} \\ 4 \\ \\ -11 \end{array}\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -7 & -6 & +8 \\ 0 & \boxed{-4} & -1 & -1 & -7 \end{array}\right] \xrightarrow[R_4 \to -11R_4]{R_2 \to 4R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -44 & 36 & 24 & -136 \\ 0 & 0 & -7 & -6 & 8 \\ 0 & \boxed{44} & 11 & 11 & 77 \end{array}\right] \xrightarrow[R_2 + R_4]{R_4 \to}$$

$$\begin{array}{c} \\ \\ 47 \\ 7 \end{array}\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -7 & -6 & 8 \\ 0 & 0 & \boxed{47} & 35 & -59 \end{array}\right] \xrightarrow[R_4 \to 7R_4]{R_3 \to 47R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -329 & -282 & 376 \\ 0 & 0 & \boxed{329} & 245 & -413 \end{array}\right] \xrightarrow[R_4 \to R_3 + R_4]{}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -7 & -6 & 8 \\ 0 & 0 & 0 & -37 & -37 \end{array}\right]$$

Using back substitution:

$-37w = -37 \Rightarrow w = 1$

$-7z - 6w = 8 \Rightarrow -7z - 6(1) = 8$

$-7z = 14$

$z = -2$

$-11y + 9z + 6w = -34 \Rightarrow$

$-11y + 9(-2) + 6(1) = -34$

$-11y - 18 + 6 = -34$

$-11y = -34 + 12$

$-11y = -22 \Rightarrow y = 2$

$x + 3y - 2z - w = 9 \Rightarrow$

$x + 3(2) - 2(-2) - 1 = 9$

$x + 9 = 9 \Rightarrow x = 0$

The solution is $(0, 2, -2, 1)$

(8) $\dim A = 3\times 3$
$\dim B = 3\times 3$
so all operations are defined

(a) $2A+B=$

$$= 2\begin{pmatrix} -3 & 1 & 1 \\ 1 & -1 & 2 \\ 5 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 2 & 2 \\ 2 & -2 & 4 \\ 10 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 8 & 3 \\ 4 & -2 & 5 \\ 10 & 1 & -1 \end{pmatrix}$$

(b) $AB = \begin{pmatrix} -3 & 1 & 1 \\ 1 & -1 & 2 \\ 5 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

$$= \begin{pmatrix} -1 & -17 & -3 \\ -1 & 8 & -2 \\ 5 & 30 & 5 \end{pmatrix}$$

(c) $B^2 = \begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 13 & 7 & 6 \\ 2 & 13 & 1 \\ 2 & -1 & 2 \end{pmatrix}$$

(9) $N(t) = 1200e^{0.57t}$

$\begin{cases} t = \text{number of hours} \\ N(t) = \text{number of bacteria} \end{cases}$

(a) $N(0) = 1200e^{0.57(0)} = 1200$

$\boxed{N(0) = 1200 \text{ bacteria}}$

(b) $N(3) = 1200e^{0.57(3)} \approx 6634$

$\boxed{N(3) = 6634 \text{ bacteria}}$

(c) find t if $N(t) = 20{,}000$

solve: $1200e^{0.57t} = 20{,}000$

$$e^{0.57t} = \frac{20{,}000}{1200} = \frac{50}{3}$$

$$\ln e^{0.57t} = \ln\frac{50}{3}$$

$$0.57t = \ln\frac{50}{3}$$

$$t = \frac{\ln\frac{50}{3}}{0.57} \approx 4.94 \text{ hours}$$

$\boxed{N(4.94) = 20{,}000 \text{ bacteria}}$

(d) find t such that $N(t) = 2(1200)$

solve: $2(1200) = 1200e^{0.57t}$

$$2 = e^{0.57t}$$

$$\ln 2 = \ln e^{0.57t}$$

$$\ln 2 = 0.57t$$

$$t = \frac{\ln 2}{0.57} \approx 1.22 \text{ hours}$$

it takes about 1.22 hours for the population to double

(e) let $N = 1200e^{0.57t}$

solve for t

$$e^{0.57t} = \frac{N}{1200}$$

$$\ln e^{0.57t} = \ln \frac{N}{1200}$$

$$0.57t = \ln \frac{N}{1200}$$

$$\boxed{t = \frac{1}{0.57} \ln \frac{N}{1200}}$$

While the function $N = 1200e^{0.57t}$ gives the number of bacteria at a given time t (in hours), the inverse function

$$t = \frac{100}{57} \ln \frac{N}{1200}$$

gives the number of hours it take the population to become N bacteria.

(10) (a) (1) let $y = \frac{3x+2}{4x-1}$

(2) solve for x

$$y(4x-1) = 3x+2$$

$$4xy - y = 3x+2$$

$$4xy - 3x = 2+y$$

$$x(4y-3) = 2+y$$

$$x = \frac{2+y}{4y-3}$$

(3) $x \leftrightarrow y$

$$y = \frac{2+x}{4x-3}, \text{ so } \boxed{f^{-1}(x) = \frac{2+x}{4x-3}}$$

(b) (1) let $y = \log_2\left(\frac{x-1}{x+3}\right)$

(2) solve for x:

$$2^y = \frac{x-1}{x+3}$$

$$2^y(x+3) = x-1$$

$$2^y \cdot x + 3\cdot 2^y = x-1$$

$$3\cdot 2^y + 1 = x - 2^y \cdot x$$

$$3\cdot 2^y + 1 = x(1-2^y)$$

$$x = \frac{3\cdot 2^y+1}{1-2^y}$$

(3) $x \leftrightarrow y$

$$y = \frac{3\cdot 2^x+1}{1-2^x}$$

$$\boxed{g^{-1}(x) = \frac{3\cdot 2^x+1}{1-2^x}}$$

(c) (1) let $y = 4\sqrt[3]{x+5}$

(2) solve for x

$y^3 = \left(4\sqrt[3]{x+5}\right)^3$

$y^3 = 64(x+5)$

$x+5 = \frac{y^3}{64}$

$x = \frac{y^3}{64} - 5$

(3) $x \Longleftrightarrow y$

$y = \frac{x^3}{64} - 5$

$$\boxed{h^{-1}(x) = \frac{x^3}{64} - 5}$$

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(11) given

n = # of vibrations per sec.

T = tension

l = length of string

$n = k \cdot \frac{\sqrt{T}}{l}$

$n = 5$ when $T = 225$, $l = 0.6$

find n when $T = 196$, $l = 0.65$

Solution

$n = \frac{k\sqrt{T}}{l}$, $k = ?$

$5 = \frac{k \cdot \sqrt{225}}{0.6}$

$5 = \frac{15k}{0.6}$

$0.6(5) = 15k \Rightarrow$

$k = \frac{0.6(5)}{15} = \frac{0.6}{3} = 0.2$

so $n = \frac{0.2\sqrt{T}}{l}$

$\therefore n = \frac{0.2\sqrt{196}}{0.65} = \frac{0.2(14)}{0.65} \approx 4.3$

The number of vibrations per second is about 4.3

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(12) (a) We'll use

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

where $A = 30{,}000$

$r = 0.04$

$n = 12$

$t = 8$

find P

Solution

$30{,}000 = P\left(1 + \frac{0.04}{12}\right)^{12(8)}$

$30{,}000 = P\left(\frac{12.04}{12}\right)^{96}$

$P = \frac{30{,}000}{\left(\frac{12.04}{12}\right)^{96}} \approx 21{,}796$ \$

We need to deposit a minimum of 21,796 \$ at 4% to have 30,000 \$ after 8 years.

(b) we'll use

$$A = Pe^{rt}$$

where $P = 5000$

$A = 30{,}000$

$r = 0.04$

find t

Solution

$$30{,}000 = 5000\, e^{0.04t}$$

$$e^{0.04t} = \frac{30{,}000}{5000} = 6$$

$$\ln e^{0.04t} = \ln 6$$

$$0.04t = \ln 6$$

$$t = \frac{\ln 6}{0.04} \approx 44.8 \text{ years}$$

it will take about 44.8 years for the 5000 \$ deposited at 4% compounded continuously to grow to 30,000 \$