Math 130 Winter 2012 Name:

QUIZ #2 @ 115 points

Write neatly. Show all work. **Write all exercises on separate paper. Clearly label the exercises.**

1. Identify all the asymptotes of the following functions:

a) $f(x)=\dfrac{8x^2+2x-10}{2x^2-3x-5}$ b) $g(x)=\dfrac{2x^2+5}{x-3}$ c) $h(x)=\dfrac{x^2-4}{x^2+3x+2}$

d) $l(x)=\dfrac{4x^2+25}{x^2+9}$ e) $F(x)=\dfrac{2x-3}{x^2-1}$

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2. Let $f(x)=\dfrac{x+1}{x-2}$.

a) Graph the function showing all work: asymptotes, intercepts, test points(if needed).

b) Find the inverse function.

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3. Let $f(x)=e^{x+3}-2$.

a) Graph the function using transformations. Clearly show how you are obtaining the graph, that is, show all equations, their meaning, and the corresponding graphs.

b) State the domain, range, and asymptote.

c) Does the function have an inverse? Explain.

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4. Consider $f(x)=\dfrac{9x^2-1}{x^2-4}$.

Questions $a-f$ below relate to this polynomial function.

a) Factor the numerator and the denominator.

b) What is the domain of the function?

c) What are the vertical asymptotes?

d) What is the horizontal asymptote?

e) What are the intercepts for this function? Write them as ordered pairs.

f) Plot additional points (if necessary) to get the shape of this function and sketch a graph.

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5. Find the inverse of each function:

a) $f(x)=\dfrac{x-1}{x-2}$ b) $g(x)=\sqrt{x+1}$

Extra credit @ 6 points

Suppose a population is described by the equation $P=f(t)=20+0.4t$, where P is the number of people (in thousands) and t is the number of years since 1970.

a) What was the population in 1970?

b) Evaluate $f(25)$. Explain in words what this tells you about the population.

c) Evaluate $f^{-1}(25)$. Explain in words what this tells you about the population.

Quiz 2 - Solutions

① (a) $f(x) = \dfrac{8x^2+2x-10}{2x^2-3x-5}$

$8x^2+2x-10 = 2(4x^2+x-5)$
$= 2(4x+5)(x-1)$

$2x^2-3x-5 = (2x-5)(x+1)$

so $f(x) = \dfrac{2(4x+5)(x-1)}{(2x-5)(x+1)}$

Domain: $x \neq \frac{5}{2}$, $x \neq -1$

V.A. $\begin{cases} x = \frac{5}{2} \\ x = -1 \end{cases}$

H.A. $y = \frac{8}{2} = 4$

O.A. not applicable

(b) $g(x) = \dfrac{2x^2+5}{x-3}$

Domain: $x \neq 3$

V.A. $x = 3$
H.A. none

O.A.

$$\begin{array}{c|ccc} & 2 & 0 & 5 \\ \hline 3 & 2 & 6 & 23 \end{array} \; R$$

$\dfrac{2x^2+5}{x-3} = 2x+6+\dfrac{33}{x-3}$

so O.A. $y = 2x+6$

(c) $h(x) = \dfrac{x^2-4}{x^2+3x+2}$

$h(x) = \dfrac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+1)} = \dfrac{x-2}{x+1}$

Domain: $x \neq -2$, $x \neq -1$

V.A. $x = -1$
H.A. $y = \frac{1}{1} = 1$
O.A. none

(d) $\ell(x) = \dfrac{4x^2+25}{x^2+9}$

Domain: $x \in \mathbb{R}$

V.A. none
H.A. $y = \frac{4}{1} = 4$
O.A. none

(e) $F(x) = \dfrac{2x-3}{x^2-1} = \dfrac{2x-3}{(x-1)(x+1)}$

Domain: $x \neq \pm 1$

V.A. $x = 1$
$x = -1$

H.A. $y = 0$

O.A. none

(2) $f(x) = \dfrac{x+1}{x-2}$

1. Domain $x \neq 2$
2. V.A. $x = 2$
   H.A. $y = 1$
3. x-n: $y=0$ iff $x+1=0$
   $x=-1$
   $\boxed{(-1,0)}$

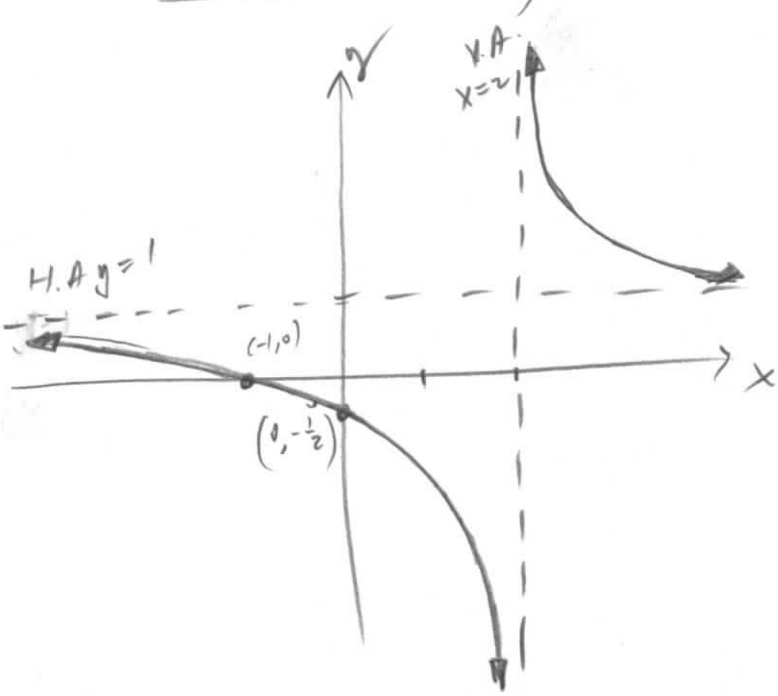
   y-n: $x=0$, $y=\frac{-1}{2}$
   $\boxed{(0,\frac{-1}{2})}$
4. Check if $f(x)$ intersects H.A. $y=1$

$f(x) \stackrel{?}{=} 1$

$\frac{x+1}{x-2} = 1$ iff

$x+1 = x-2$ Contradiction

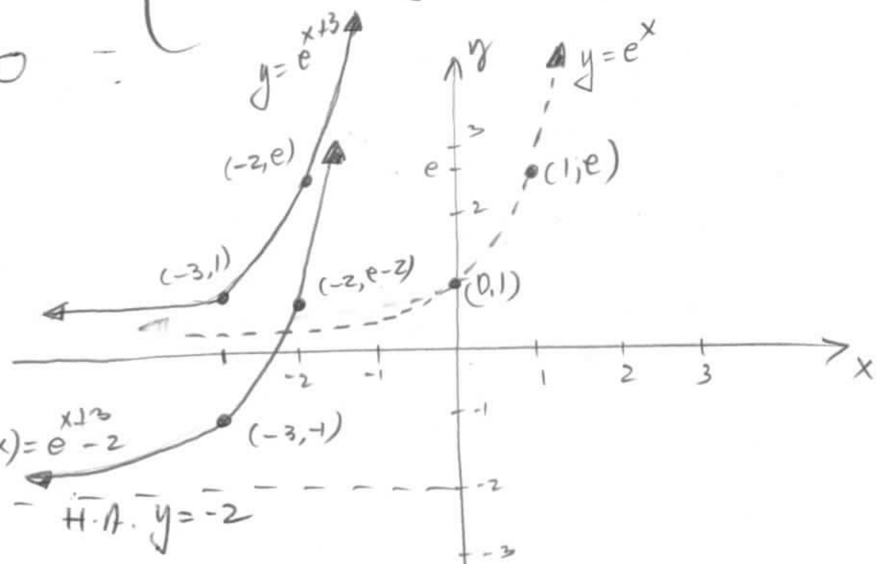
So no common points.



(3) $f(x) = e^{x+3} - 2$

x	y
0	1
1	e

(a) 1st let $y=e^x$
2nd $y=e^{x+3}$ shift left 3
3rd $y=e^{x+3}-2$ shift down 2



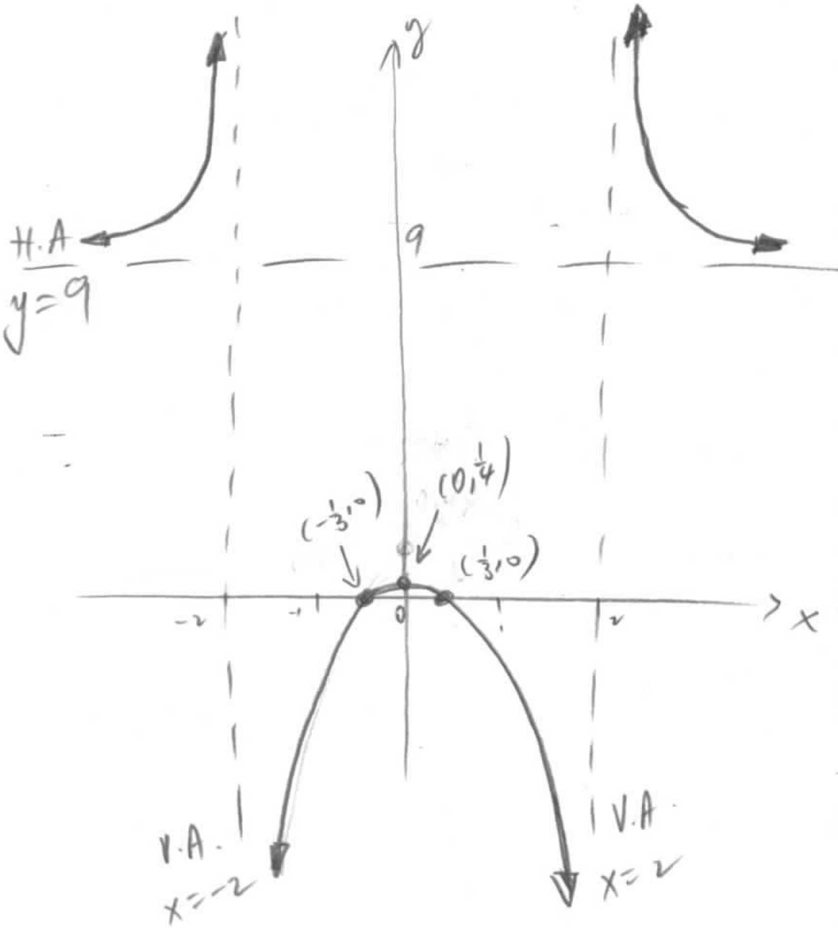
(b) Domain $x \in \mathbb{R}$
Range $y > -2$
H.A. $y = -2$

(c) Yes, the function has an inverse because it is one-to-one, as its graph passes the Horizontal Line Test.

(4) $f(x) = \dfrac{9x^2-1}{x^2-4}$

(a) $9x^2-1 = (3x)^2 - 1^2$
$= (3x-1)(3x+1)$
$x^2-4 = x^2 - 2^2$
$= (x-2)(x+2)$

so $f(x) = \dfrac{(3x-1)(3x+1)}{(x-2)(x+2)}$



(b) Domain: $x \neq 2,\ x \neq -2$

(c) V.A. $x = 2,\ x = -2$

(d) H.A. $y = 9$

(e) x-n: $y = 0$ iff $(3x-1)(3x+1) = 0$
$x = \frac{1}{3},\ x = -\frac{1}{3}$

$\boxed{(\frac{1}{3}, 0), (-\frac{1}{3}, 0)}$

y-n: $x = 0,\ y = \frac{-1}{-4} = \frac{1}{4}$

$\boxed{(0, \frac{1}{4})}$

(f) Test Point: $x = -3$,
$y = \dfrac{9(-3)^2-1}{(-3)^2-4} > 0$

Test Point: $x = 3$
$y = \dfrac{9(3)^2-1}{3^2-4} > 0$

(5) (a) $f(x) = \dfrac{x-1}{x-2}$

1st $y = \dfrac{x-1}{x-2}$

2nd solve for x
$y(x-2) = x-1$
$yx - 2y = x-1$
$yx - x = 2y - 1$
$x(y-1) = 2y-1$
$x = \dfrac{2y-1}{y-1}$

3rd $x \longleftrightarrow y$

$y = \dfrac{2x-1}{x-1}$

so $\boxed{f^{-1}(x) = \dfrac{2x-1}{x-1}}$

(b) $g(x) = \sqrt{x+1}$

1st $y = \sqrt{x+1}$

2nd solve for x

$y^2 = x+1$

$x = y^2 - 1$

3rd $x \longleftrightarrow y$

$y = x^2 - 1$

so $\boxed{g^{-1}(x) = x^2 - 1}$

EXTRA CREDIT

$P = f(t) = 20 + 0.4t$

t = number of years since 1970

$f(t) = P$ = number of people (in thousands)

(a) $f(0) = 20$ thousand people

(b) $f(25) = 20 + 0.4(25)$
$= 30$ thousand people

$f(25)$ gives the population (in thousands) 25 years after 1970; that is, there were 30 thousand people in 1995.

(c) $f^{-1}(25) = ?$

We know that

$f(a) = b \iff f^{-1}(b) = a$

So, $f^{-1}(25) = x$ iff

$f(x) = 25$

Solve $f(x) = 25$

$20 + 0.4t = 25$

$0.4t = 5$

$t = \frac{5}{0.4} = \frac{50}{4} = 12.5$

so $f^{-1}(25) = 12.5$ years after 1970

$f^{-1}(25)$ gives the number of years it takes after 1970 for the population to be 25 thousand people.