Math 71X 2010 Name:

TEST 1 @ 200 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. No proof, no credit given!

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1. Suppose that 35 gallons of gas cost \$95. How many liters can be purchased for \$63 ?

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2. Suppose that an 145 lb patient received 12 mg of a particular medication. If the medication to weight rate is preserved, what dose should be appropriate for a patient weighing 90 kg? Round the answer to four significant digits.

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3. Evaluate and express in scientific notation, rounded appropriately: $\sqrt{\dfrac{21.43\times10^{-11}}{3.193\times10^{-25}}}$.

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4. Simplify the following. Your answers should have only positive exponents. Assume all variable under the square root are positive.

a) $\dfrac{2x^2y^3}{15}+\dfrac{2}{3}x^2y^3$ b) $\sqrt{\dfrac{x^3y^{-4}}{x^{-5}y^8}}$ c) $4\sqrt{20}-3\sqrt{45}$

d) $\left(ab^{-3}\right)^4\left(a^2b^{-1}\right)^{-2}$ e) $\dfrac{3cd+6cx}{9fd+18fx}$

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5. Solve the following equations:

a) $x-2\left(\dfrac{1}{3}x+\dfrac{2}{5}\right)=\dfrac{2}{15}-\dfrac{1}{5}x$ b) $2\left|x-1\right|=5$ c) $W=\dfrac{1}{2}m\left(v_2{}^2-v_1{}^2\right)$ solve for m

d) $C=\dfrac{5}{9}\left(F-32\right)$ solve for F e) $T=2p\sqrt{\dfrac{L}{g}}$ solve for L f) $\dfrac{1}{2}gt^2=y$ solve for t

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6. Solve the following inequalities. Graph the solution set and write it using interval notation.

a) $\dfrac{1}{4}-\dfrac{1}{2}\left(y-3\right)\geq\dfrac{3}{2}y+\dfrac{3}{4}$ b) $3\left|x+5\right|<15$ c) $2+\left|2x-1\right|\geq9$

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7. Let $f\left(x\right)=-2x+4$. Do the following:

a) Graph the function .
b) State the domain and range.
c) Find the intercepts.
d) What is the slope of the line?
e) What is the slope of a line perpendicular to this line?
f) Find $f\left(-\dfrac{1}{2}\right)$ and $f\left(a+b\right)$.

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8. Find an equation of the line passing through $(-1,3)$ and $(-2,-8)$.

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9. In an anthropological study of fossils, researcher's objective was to predict age (y) from the percentage (x) of a tooth's root with transparent dentine.

x	y
15	23
31	65
41	32
47	78
55	61
65	60

a) Create a scatter-plot for these values.

b) From the scatter-plot, pick two points which represent the linear trend amongst the data points and determine the equation of the line which contains these points.

c) Use the equation of this line to predict the age when 60% of a tooth's root has transparent dentine.

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10. Suppose a particular medication is to be administered to a patient at a rate of 250 mg per 100 lb. If a patient's weigh is estimated at 170 lb with a maximum error of 5 lb, what is the patient's dosage and the corresponding maximum error and maximum relative error in the computed dosage?

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11. A company has a monthly fixed cost of $12,650 and produces cell phone for $97 per unit.

a) Create a linear function which gives the total cost required to produce x phones.

b) What is the total cost of producing 55 units?

d) How many units can be produced for 50,000 ?

TEST 1 - SOLUTIONS

(1) 35 gal ... 95 ¢
? L ... 63 ¢

Solution

$$63 \cancel{¢} \cdot \frac{35\,\cancel{gal}}{95\,\cancel{¢}} \cdot \frac{3.7854L}{1\,\cancel{gal}} =$$

$$= \frac{63 \cdot 35 \cdot 3.7854}{95} L = 87.8611 L$$

$$\approx \boxed{87.9 L}$$

(2) 145 lb .... 12 mg
90 kg .... ? mg

Solution

$$90\,\cancel{kg} \cdot \frac{2.2046\,\cancel{lb}}{1\,\cancel{kg}} \cdot \frac{12\,mg}{145\,\cancel{lb}} =$$

$$= \frac{90 \cdot 2.2046 \cdot 12}{145} mg = 16.4205\,mg$$

$$\approx \boxed{16.42 mg}$$

(3) $$\sqrt{\frac{21.43 \times 10^{-11}}{3.193 \times 10^{-25}}} = \sqrt{\frac{21.43}{3.193} \times 10^{14}}$$

$$= \sqrt{\frac{21.43}{3.193}} \times 10^{7}$$

$$= 2.59067 \times 10^{7}$$

$$\approx 2.591 \times 10^{7}$$

scientific notation

(4) (a) $$\frac{2x^2y^3}{15} + \frac{2}{3}x^2y^3 =$$

$$= \left(\frac{2}{15} + \frac{{}^{5/}2}{3}\right) x^2y^3 = \frac{12}{15}x^2y^3$$

$$= \boxed{\frac{4}{5}x^2y^3}$$

(b) $$\sqrt{\frac{x^3y^{-4}}{x^{-5}y^8}} = \sqrt{x^8y^{-12}} = \sqrt{\frac{x^8}{y^{12}}}$$

$$= \boxed{\frac{x^4}{y^6}}$$

(c) $$4\sqrt{20} - 3\sqrt{45} =$$

$$= 4\sqrt{4 \cdot 5} - 3\sqrt{9 \cdot 5}$$

$$= 4 \cdot 2\sqrt{5} - 3 \cdot 3\sqrt{5} = \boxed{-\sqrt{5}}$$

(d) $$(ab^{-3})^4(a^2b^{-1})^{-2} =$$

$$= a^4b^{-12}a^{-4}b^2 = a^0b^{-10} = \boxed{\frac{1}{b^{10}}}$$

(e) $$\frac{3cd + 6cx}{9fd + 18fx} = \frac{3c(d+2x)}{9f(d+2x)}$$

$$= \boxed{\frac{c}{3f}}$$

(5) (a) $$x - 2\left(\frac{1}{3}x + \frac{2}{5}\right) = \frac{2}{15} - \frac{1}{5}x$$

$$^{15/}x - \frac{{}^{5/}2}{3}x - \frac{{}^{3/}4}{5} = \frac{2}{15} - \frac{{}^{3/}1}{5}x$$

LCD = 15

$$15x - 10x - 12 = 2 - 3x$$

$$5x - 12 = 2 - 3x$$

$$5x + 3x = 2 + 12$$

$$8x = 14$$

$$x = \frac{14}{8} = \frac{7}{4}$$

$$\boxed{x \in \left\{\frac{7}{4}\right\}}$$

(b) $$2|x-1| = 5$$

$$|x-1| = \frac{5}{2}$$

$$x - 1 = \frac{5}{2} \quad \text{OR} \quad x - 1 = \frac{-5}{2}$$

$$x = {}^{2/}1 + \frac{5}{2} \qquad x = {}^{2/}1 - \frac{5}{2}$$

$$x = \frac{7}{2} \qquad x = \frac{-3}{2}$$

$$\boxed{x \in \left\{\frac{7}{2}, \frac{-3}{2}\right\}}$$

(c) $W = \frac{1}{2} m (V_2^2 - V_1^2)$, $m = ?$

$2W = m(V_2^2 - V_1^2)$

$$\boxed{m = \frac{2W}{V_2^2 - V_1^2}}$$

(d) $C = \frac{5}{9}(F - 32)$, $F = ?$

$9C = 5(F - 32)$

$F - 32 = \frac{9C}{5}$

$$\boxed{F = 32 + \frac{9C}{5}}$$

(e) $T = 2\pi\sqrt{\frac{L}{g}}$, $L = ?$

$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$ $\quad /^2$

$\frac{T^2}{4\pi^2} = \frac{L}{g}$

$T^2 g = L \cdot 4\pi^2 \Rightarrow \boxed{L = \frac{T^2 g}{4\pi^2}}$

(f) $\frac{1}{2} g t^2 = y$, $t = ?$

$g t^2 = 2y$ $\quad /\sqrt{\ }$

$t^2 = \frac{2y}{g}$

$\sqrt{t^2} = \sqrt{\frac{2y}{g}}$

$t = \pm\sqrt{\frac{2y}{g}}$

$$\boxed{t = \sqrt{\frac{2y}{g}}}$$

(6) (a) $\frac{1}{4} - \frac{1}{2}(y-3) \geq \frac{3}{2}y + \frac{3}{4}$

LCD = 4

$1 - 2(y-3) \geq 6y + 3$

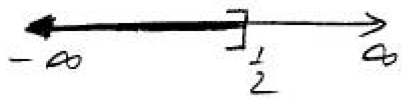
$1 - 2y + 6 \geq 6y + 3$

$7 - 2y \geq 6y + 3$

$7 - 3 \geq 6y + 2y$

$4 \geq 8y$

$\frac{4}{8} \geq y \Rightarrow \boxed{y \leq \frac{1}{2}}$



$y \in (-\infty, \frac{1}{2}]$

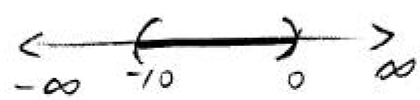
(b) $3|x+5| < 15$

$|x+5| < 5$

$-5 < x+5 < 5$

$-5-5 < x < 5-5$

$$\boxed{-10 < x < 0}$$



$x \in (-10, 0)$

(c) $2 + |2x-1| \geq 9$

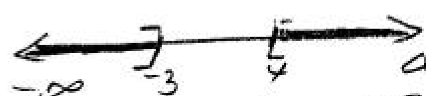
$|2x-1| \geq 7$

$2x - 1 \leq -7$ OR $2x - 1 \geq 7$

$2x \leq -6$ $\quad$ $2x \geq 8$

$x \leq -3$ $\quad$ $x \geq 4$

$$\boxed{x \leq -3 \text{ OR } x \geq 4}$$



$x \in (-\infty, -3] \cup [4, \infty)$

(7) $f(x) = -2x + 4$

(a) The graph is a line

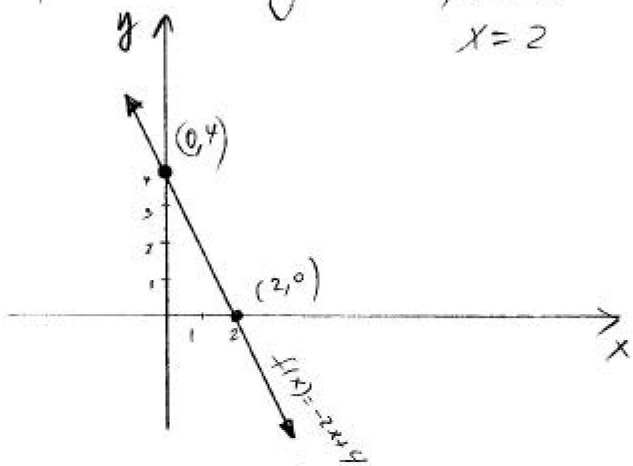
x	y
0	4
2	0

when $x = 0$, $y = 4$

when $y = 0$, $-2x + 4 = 0$

$4 = 2x$

$x = 2$



(b) Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$

(c) x-int: (2,0)

y-int: (0,4)

(d) $m = -2$ ($y = -2x + 4$ is the slope-intercept form)

(e) $m_{\perp} = \frac{+1}{2}$

(f) $f\left(\frac{-1}{2}\right) = -2\left(\frac{-1}{2}\right) + 4 = 1 + 4$

$f\left(\frac{-1}{2}\right) = 5$

$f(a+b) = -2(a+b) + 4$

$f(a+b) = -2a - 2b + 4$

(8) $(-1, 3)$ and $(-2, -8)$

$m = \frac{\Delta y}{\Delta x} = \frac{3-(-8)}{-1-(-2)} = \frac{11}{1} = 11$

Use $m = 11$ and $(-1, 3)$

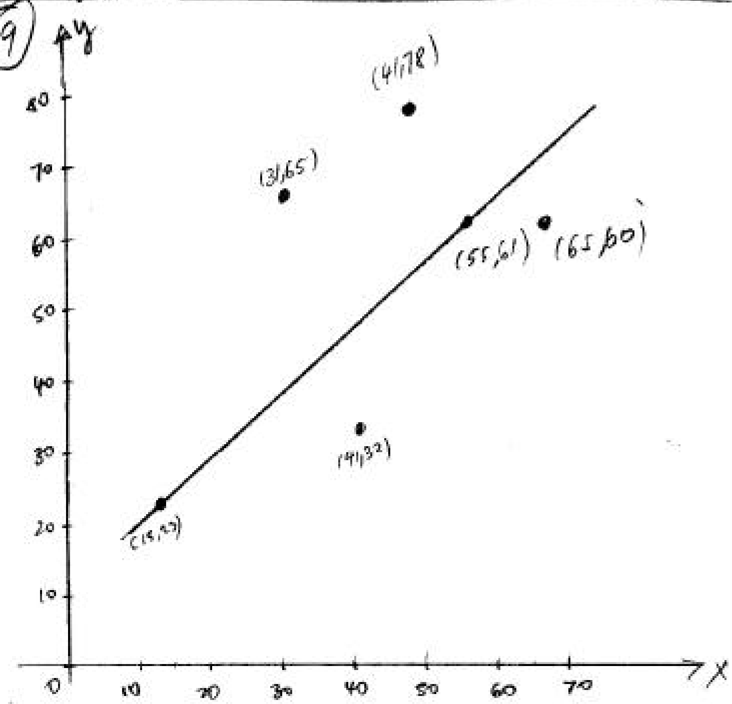
$y - y_1 = m(x - x_1)$

$\boxed{y - 3 = 11(x + 1)}$

or

$y = 11x + 14$

(9)



(b) The points chosen are $(15, 23)$ and $(55, 61)$

$m = \frac{\Delta y}{\Delta x} = \frac{61-23}{55-15} = \frac{38}{40} = 0.95$

Use $m = 0.95$ and $(15, 23)$

$y - y_1 = m(x - x_1)$

$y - 23 = 0.95(x - 15)$ or

$y = 0.95x + 8.75$

(c) $x = 60$ percent, then

$y = 0.95(60) + 8.75$

$y = 65.75 \approx 65.8$ years.

(10) $\left[\begin{array}{l} 250 \text{ mg per } 100 \text{ lb} \\ 170 \text{ lb patient} \\ \text{error} = 5 \text{ lb} \end{array}\right.$

Find $\left\{\begin{array}{l} \text{dosage} \\ \text{max. error} \\ \text{max. rel. error} \end{array}\right.$

Solution

let $y =$ dosage (mg)
$x =$ weight (lb)

$y = mx$, where $m =$ rate of change of dosage (y) with respect to weight (x)

so $m = \dfrac{250 \text{ mg}}{100 \text{ lb}} = 2.5 \dfrac{\text{mg}}{\text{lb}}$

So $y = 2.5x$

when $x = 170$ lb

$y = 2.5 \dfrac{\text{mg}}{\cancel{\text{lb}}} \cdot 170 \cancel{\text{lb}}$

$\boxed{y = 425 \text{ mg}}$ dosage

$\Delta y = m \, \Delta x$

$\Delta y = 2.5 \, \Delta x$

when $\Delta x = 5$ lb,

$\Delta y = 2.5 \dfrac{\text{mg}}{\cancel{\text{lb}}} \cdot 5 \cancel{\text{lb}}$

$\boxed{\Delta y = 12.5 \text{ mg}}$ max. error

max. rel. error $= \dfrac{\Delta y}{y} \cdot 100\%$

$= \dfrac{12.5 \text{ mg}}{425 \text{ mg}} \cdot 100\%$

$= 2.94118\%$

$\approx \boxed{2.94\%}$ max. rel. error

(11) mo. fixed costs $= 12{,}650$ \$

$97 \dfrac{\$}{\text{unit}}$

(a) let $T(x) =$ total cost of producing x units

$\boxed{T(x) = 12{,}650 + 97x}$!

(b) when $x = 55$ units,

$T(55) = 12{,}650 + 97(55)$

$T(55) = 17{,}985$ \$

(c) $x = ?$ if $T(x) = 50{,}000$ \$

$12{,}650 + 97x = 50{,}000$

$97x = 50{,}000 - 12{,}650$

$97x = 37{,}350$

$x = \dfrac{37{,}350}{97} = 385.052 \approx 385$ units

385 units can be produced for 50,000 \$.