Math 180 B Fall 2010 Name:

TEST #1 @ 180 points

Write neatly. Show all work. **Write all proofs on separate paper. Label each exercise.**

1. a) Explain the meaning of: $\lim_{x\to a} g(x) = L$.

   b) Write the mathematical definition of: *The function* $y = g(x)$ *is continuous at* $x = a$.

   c) When is a function $y = f(x)$ discontinuous at $x = a$?

   d) Write the mathematical definition of : *The function* $y = f(x)$ *is differentiable at* $x = c$.

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2. Find the following limits. If a limit does not exist, explain why.

   a) $\lim_{x\to 0} x^6 \sin\frac{2}{x}$.   b) $\lim_{t\to\infty}\frac{\sin t}{t}$ c) $\lim_{y\to 0}\frac{\cos y - 1}{y}$

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3. Prove the following Theorem: *If a function f is differentiable at a point c, then it is continuous at c.*

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4. Prove the Derivative Difference Formula:

   If g and *l* are differentiable functions of *x*, then their difference $g - l$ is differentiable at every point where *g* and *l* are both differentiable. At such points,

$$(g-l)'(x) = g'(x) - l'(x)$$

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5. Let $f(x) = \begin{cases} 4x - 1, x \le a \\ x^2 + 1, x > a \end{cases}$ Find a such that *f* is continuous everywhere.

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6. Find an equation of the tangent line to the graph of $f(x) = xe^x$ when $x = 2$.

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7. Find the derivative of each function in two ways: i) using the definition; ii) using the differentiation rules.

   a) $f(x) = \frac{x}{x-1}$ b) $g(x) = 2 + \sqrt{x}$

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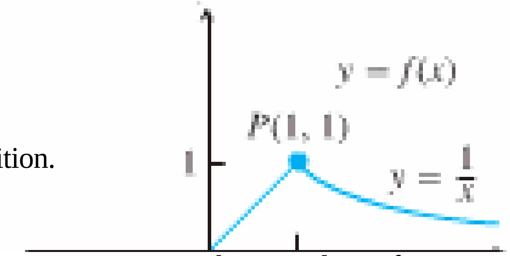
8. Complete the following <u>. Do not prove</u>. If a limit does not exist, say so, and explain why.

   a) $\lim_{t\to 0}\frac{\sin t}{t} =$ c) $\lim_{x\to 0^+}\frac{1}{x} =$ e) $\lim_{x\to\infty} e^x =$

   b) $\lim_{q\to\infty}\frac{1}{q} =$ d) $\lim_{a\to 0^-}\frac{1}{a} =$ f) $\lim_{x\to -\infty} e^x =$

9. Use the following graph to answer the questions.
<u>If a limit does not exist, explain.</u>



a) $\lim_{x\to 1} f(x)$ b) $\lim_{x\to 0} f(x)$ c) $\lim_{x\to 3} f(x)$

d) Is the function f continuous at $x=1$? Explain using the definition.

e) Is the function f differentiable at $(1,1)$? Do not just write an answer. Write a mathematical proof.

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10. Find the following limits. Do not just write an answer, show proof.

a) $\lim_{x\to\infty} \frac{3x^7-2x^4+3x+1}{5x^7+x^2-x-4}$ d) $\lim_{x\to 0} \cos\left(\frac{p}{2}\cos(\tan x)\right)$ f) $\lim_{y\to 0} \frac{y\csc 3y}{\cos 4y}$

b) $\lim_{t\to\infty} \frac{2t+\cos t-4\sqrt{t}}{3t+\cos t}$ e) $\lim_{x\to 0} \frac{x^2+x}{x^5+2x^4+x^3}$ g) $\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x}$

c) $\lim_{x\to\infty} \left(x-\sqrt{x^2-1}\right)$

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11. Find the derivative of each function.

a) $y=4x^3+3x-5$ b) $y=5\sqrt[3]{x}+4xe^x$ c) $y=\frac{2x-7}{x^2+1}$

d) $y=\frac{t^2}{4t^2-3t+2}$ e) $y=\frac{(x-1)(x^2-2x)}{x^4}$

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12. The function $f(t)=16t^2$ gives the distance (in feet) of a rock falling freely during the first t seconds.

a) Find the average speed of the rock between $t=1$ s and $t=4$ s.

b) Find the instantaneous velocity at exactly $t=3$ s.

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13. Find equations of both lines through the point $(2,-3)$ that are tangent to the parabola $y=x^2+x$.

EXTRA CREDIT – You can choose a maximum of THREE extra credit questions.

#1 @ 2 points: Can $f(x)=\dfrac{x(x^2-1)}{|x^2-1|}$ be extended to be continuous at $x=1$ or -1? Give reasons for your answers.

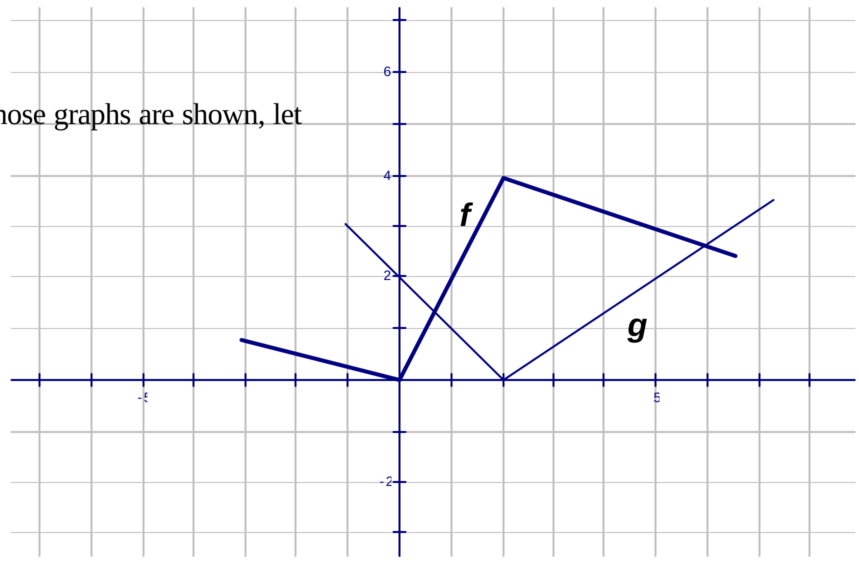
#2 @ 2 points If $P=\dfrac{nRT}{V-nb}-\dfrac{an^2}{V^2}$, find $\dfrac{dP}{dV}$.

#3 @ 3 points Prove the Derivative Product Rule .

#4 @ 4 points If f and g are the functions whose graphs are shown, let

$u(x)=f(x)g(x)$ and $v(x)=\dfrac{f(x)}{g(x)}$.

a) Find $u'(1)$. b) Find $v'(5)$.



#5 @ 5 points Prove that $\lim_{q\to 0^+}\dfrac{\sin q}{q}=1$.

TEST 1B - SOLUTIONS

① (a) $\lim_{x\to a} g(x) = L$ iff the values of $g(x)$ get arbitrarily close to L by taking x sufficiently close to a, on either side of a, but not equal to a.

(b) g is continuous at $x=a$ iff $\lim_{x\to a} g(x) = g(a)$

(c) f is discontinuous at $x=a$ iff $f(a)$ is not defined

or

$\lim_{x\to a} f(x)$ does not exist

or

$\lim_{x\to a} f(x) \neq f(a)$

(d) f is differentiable at $x=c$ iff $f'(c)$ exists, that is if

$$\lim_{x\to c} \frac{f(x)-f(c)}{x-c} = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h} = f'(c)$$

exists.

② (a) $\lim_{x\to 0} x^6 \sin \frac{2}{x} = ?$

Solution

Note that the limit laws cannot be applied, as $\lim_{x\to 0} \sin\frac{2}{x}$ does not exist.

We know $-1 \le \sin\frac{2}{x} \le 1$, $\forall x \neq 0$

When $x \to 0$, $x^6 > 0$, so

$$-x^6 \le x^6 \sin\frac{2}{x} \le x^6, \ \forall x \neq 0$$

$$\lim_{x\to 0} (-x^6) = \lim_{x\to 0} x^6 = 0$$

So, by the Sandwich theorem

$$\Rightarrow \lim_{x\to 0} x^6 \sin\frac{2}{x} = 0$$

(b) $\lim_{t\to\infty} \frac{\sin t}{t} = ?$

Solution

Note that $\lim_{t\to\infty} \sin t$ does not exist

We know $-1 \le \sin t \le 1$, $\forall t$

As $t\to\infty$, $\frac{1}{t} \to 0$, $\frac{1}{t} > 0$

So, $-\frac{1}{t} \le \frac{\sin t}{t} \le \frac{1}{t}$, $\forall t \neq 0$

$$\lim_{t\to\infty} \left(-\frac{1}{t}\right) = \lim_{t\to\infty} \frac{1}{t} = 0$$

By the Sandwich Th $\Rightarrow \lim_{t\to\infty} \frac{\sin t}{t} = 0$

(c) $\lim_{y\to 0} \frac{\cos y - 1}{y} = ?$

Solution

Method I

$\cos y = 1 - 2\sin^2 \frac{y}{2}$

$\lim_{y\to 0} \frac{\cos y - 1}{y} = \lim_{y\to 0} \frac{-2\sin^2 \frac{y}{2}}{y}$

$= -\lim_{y\to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}} \cdot \sin \frac{y}{2}$

$= -\lim_{y\to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}} \cdot \lim_{y\to 0} \sin \frac{y}{2}$

$= -1 \cdot 0 = 0$

so $\lim_{y\to 0} \frac{\cos y - 1}{y} = 0$

Method II

$\lim_{y\to 0} \frac{\cos y - 1}{y} = \frac{0}{0}$

$\lim_{y\to 0} \frac{\cos y - 1}{y} = \lim_{y\to 0} \frac{(\cos y - 1)(\cos y + 1)}{y(\cos y + 1)}$

$= \lim_{y\to 0} \frac{\cos^2 y - 1}{y(\cos y + 1)} = \lim_{y\to 0} \frac{-\sin^2 y}{y(\cos y + 1)}$

$= -\lim_{y\to 0} \frac{\sin y \cdot \sin y}{y(\cos y + 1)}$

$= -\lim_{y\to 0} \frac{\sin y}{y} \cdot \lim_{y\to 0} \frac{\sin y}{\cos y + 1}$

$= -1 \cdot \frac{0}{2} = -1 \cdot 0 = 0$

so $\lim_{y\to 0} \frac{\cos y - 1}{y} = 0$

(3) Given f differentiable at c

Prove f continuous at c

Proof

f diff. at $c \Rightarrow f'(c)$ exists

so $f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c} \in \mathbb{R}$

we need to show that

$\lim_{x\to c} f(x) = f(c)$

Let $f(x) - f(c) = \frac{f(x) - f(c)}{x - c} \cdot (x - c)$

true $\forall x \neq c$

Then,

$\lim_{x\to c} (f(x) - f(c)) =$

$= \lim_{x\to c} \left(\frac{f(x) - f(c)}{x - c} \cdot (x - c) \right)$

$= \lim_{x\to c} \frac{f(x) - f(c)}{x - c} \cdot \lim_{x\to c} (x - c)$

$= f'(c) \cdot 0 = 0$

So, $\lim_{x\to c} (f(x) - f(c)) = 0$

$\lim_{x\to c} f(x) - \lim_{x\to c} f(c) = 0$

$\lim_{x\to c} f(x) - f(c) = 0$

$\lim_{x\to c} f(x) = f(c)$

so f is continuous at c.

(4) Given g, ℓ - diff. at x

Prove $g-\ell$ diff. at x

$(g-\ell)'(x) = g'(x) - \ell'(x)$

Proof

g diff. at $x \Rightarrow g'(x)$ exists

ℓ diff. at $x \Rightarrow \ell'(x)$ exists

so $g'(x) = \lim_{h \to 0} \frac{g(x+h)-g(x)}{h} \in \mathbb{R}$

$\ell'(x) = \lim_{h \to 0} \frac{\ell(x+h)-\ell(x)}{h} \in \mathbb{R}$

Let $F(x) = g(x) - \ell(x)$

$F'(x) = \lim_{h \to 0} \frac{F(x+h)-F(x)}{h}$

$= \lim_{h \to 0} \frac{(g(x+h)-\ell(x+h))-(g(x)-\ell(x))}{h}$

$= \lim_{h \to 0} \frac{g(x+h)-g(x)}{h} - \lim_{h \to 0} \frac{\ell(x+h)-\ell(x)}{h}$

$= g'(x) - \ell'(x) \in \mathbb{R}$

Therefore, $g-\ell$ is differentiable at x and

$(g-\ell)'(x) = g'(x) - \ell'(x)$

(5) $f(x) = \begin{cases} 4x-1, & x \le a \\ x^2+1, & x > a \end{cases}$

Solution

Note that f is continuous for $\forall x \neq a$ (any polynomial function is continuous on its domain)

f cont. at $x = a$ iff $\lim_{x \to a} f(x) = f(a)$

we need

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

$$\left.\begin{array}{l} \lim_{x \to a^-} f(x) = \lim_{x \to a^-} (4x-1) = 4a-1 \\ \lim_{x \to a^+} f(x) = \lim_{x \to a^+} (x^2+1) = a^2+1 \\ f(a) = 4a-1 \end{array}\right\}$$

Therefore, we need

$4a-1 = a^2+1$

$a^2 - 4a + 2 = 0$

$a = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$

$a = 2 \pm \sqrt{2}$

(6) $f(x) = xe^x$, $x = 2$

Solution

Equation of the tangent line when $x=2$:

$y - y_1 = m(x - x_1)$, where

$$\begin{cases} m = f'(2) \\ (x_1, y_1) = (2, 2e^2) \end{cases}$$

$f'(x) = (xe^x)' = x'e^x + x(e^x)'$

$f'(x) = e^x + xe^x$

$f'(2) = e^2 + 2e^2 \Rightarrow m = 3e^2$

$y - 2e^2 = 3e^2(x-2)$

$y = 3e^2 x - 4e^2$ equation of the tg. line when $x = 2$

(7) (a) $f(x) = \frac{x}{x-1}$

(i) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{x+h}{x+h-1} - \frac{x}{x-1}}{h}$

$= \lim_{h\to 0} \frac{\frac{(x+h)(x-1) - x(x+h-1)}{(x+h-1)(x-1)}}{h}$

$= \lim_{h\to 0} \frac{x^2 - x + hx - h - x^2 - xh + x}{h(x+h-1)(x-1)}$

$= \lim_{h\to 0} \frac{-h}{h(x+h-1)(x-1)} = \lim_{h\to 0} \frac{-1}{(x+h-1)(x-1)}$

$= \frac{-1}{(x-1)^2}$, so $f'(x) = \frac{-1}{(x-1)^2}$

(ii) $f'(x) = \frac{x'(x-1) - x(x-1)'}{(x-1)^2}$

$= \frac{x - 1 - x}{(x-1)^2} = \frac{-1}{(x-1)^2}$

so $f'(x) = \frac{-1}{(x-1)^2}$

(b) $g(x) = 2 + \sqrt{x}$

(i) $g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$

$= \lim_{h\to 0} \frac{(2 + \sqrt{x+h}) - (2 + \sqrt{x})}{h}$

$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim_{h\to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$

so $g'(x) = \frac{1}{2\sqrt{x}}$

(ii) $g(x) = 2 + x^{1/2}$

$g'(x) = 0 + \frac{1}{2} x^{\frac{1}{2} - 1} = \frac{1}{2x^{1/2}}$

$g'(x) = \frac{1}{2\sqrt{x}}$

(8) (a) $\lim_{t\to 0} \frac{\sin t}{t} = 1$

(b) $\lim_{\theta\to\infty} \frac{1}{\theta} = \frac{1}{\infty} = 0$

(c) $\lim_{x\to 0^+} \frac{1}{x} = \frac{1}{0^+} = \infty$

(d) $\lim_{a\to 0^-} \frac{1}{a} = \frac{1}{0^-} = -\infty$

(e) $\lim_{x\to\infty} e^x = e^\infty = \infty$

(f) $\lim_{x\to -\infty} e^x = e^{-\infty} = \frac{1}{\infty} = 0$

(9) (a) $\lim_{x\to 1} f(x) = 1$

(b) $\lim_{x\to 0} f(x) = \lim_{x\to 0} x = 0$

(c) $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$

(d) $\lim_{x\to 1} f(x) = f(1) = 1$, so
f is continuous at 1

(e) $f'_{left}(1) = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1}$

Method I

$= \lim_{x\to 1^-} \frac{x-1}{x-1} = \lim_{x\to 1^-} 1 = 1$

$f'_{right}(1) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$

$= \lim_{x\to 1^+} \frac{\frac{1}{x}-1}{x-1}$

$\longrightarrow$

$= \lim_{x\to 1^+} \frac{\frac{1-x}{x}}{x-1} = \lim_{x\to 1^+} \frac{-1}{x} = -1$

so $f'(1)$ does not exist,
so f not differentiable at x=1

OR

Method II $f'_{left}(1) = \frac{dy}{dx}\Big|_{x=1} =$

$= \frac{d}{dx}(x)\Big|_{x=1} = 1$

$f'_{right}(1) = \frac{dy}{dx}\Big|_{x=1} = \frac{d}{dx}\left(\frac{1}{x}\right)\Big|_{x=1}$

$= \frac{-1}{x^2}\Big|_{x=1} = -1$

(10) (a) $\lim_{x\to\infty} \frac{3x^7-2x^4+3x+1}{5x^7+x^2-x-4} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{3-\frac{2}{x^3}+\frac{3}{x^6}+\frac{1}{x^7}}{5+\frac{1}{x^5}-\frac{1}{x^6}-\frac{4}{x^7}}$

$= \frac{3-0+0+0}{5+0-0-0} = \frac{3}{5}$

(b) $\lim_{t\to\infty} \frac{2t+\cos t-4\sqrt{t}}{3t+\cos t} =$

$= \lim_{t\to\infty} \frac{2+\frac{\cos t}{t}-\frac{4}{\sqrt{t}}}{3+\frac{\cos t}{t}}$

$= \frac{2+0-0}{3+0} = \frac{2}{3}$

(c) $\lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right) = \infty-\infty$

$= \lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right)\dfrac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$

$= \lim_{x\to\infty}\dfrac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim_{x\to\infty}\dfrac{1}{x+\sqrt{x^2-1}}$

$= \dfrac{1}{\infty+\infty} = \dfrac{1}{\infty} = 0$

(d) $\lim_{x\to 0}\cos\left(\frac{\pi}{2}\cos(\tan x)\right) =$

$= \cos\left(\lim_{x\to 0}\frac{\pi}{2}\cos(\tan x)\right)$

$= \cos\left(\frac{\pi}{2}\lim_{x\to 0}\cos(\tan x)\right)$

$= \cos\left(\frac{\pi}{2}\cdot\cos 0\right) = \cos\left(\frac{\pi}{2}\right) = 0$

(cosine and tangent functions are continuous on their domain)

(e) $\lim_{x\to 0}\dfrac{x^2+x}{x^5+2x^4+x^3} = \dfrac{0}{0}$

$= \lim_{x\to 0}\dfrac{x(x+1)}{x^3(x^2+2x+1)}$

$= \lim_{x\to 0}\dfrac{x+1}{x^2(x+1)^2} = \lim_{x\to 0}\dfrac{1}{x^2(x+1)}$

$= \dfrac{1}{0^+(1)} = \dfrac{1}{0^+} = \infty$

(f) $\lim_{y\to 0}\dfrac{y\csc 3y}{\cos 4y} =$

$= \lim_{y\to 0}\dfrac{y\cdot\frac{1}{\sin 3y}}{\cos 4y}$

$= \lim_{y\to 0}\dfrac{y}{\sin 3y\cos 4y}$

$= \lim_{y\to 0}\dfrac{1}{\frac{\sin 3y}{3y}\cdot 3}\ \lim_{y\to 0}\dfrac{1}{\cos 4y}$

$= \dfrac{1}{1\cdot 3}\cdot 1 = \dfrac{1}{3}$

(g) $\lim_{x\to 4}\dfrac{2-\sqrt{x}}{4-x} = \dfrac{0}{0}$

$= \lim_{x\to 4}\dfrac{2-\sqrt{x}}{(2-\sqrt{x})(2+\sqrt{x})}$

$= \lim_{x\to 4}\dfrac{1}{2+\sqrt{x}} = \dfrac{1}{2+2} = \dfrac{1}{4}$

(11)(a) $y = 4x^3+3x-5$

$y' = \dfrac{dy}{dx} = 4\cdot 3x^2+3\cdot 1-0$

$y' = 12x^2+3$

(b) $y = 5\sqrt[3]{x}+4xe^x$

$y = 5x^{\frac{1}{3}}+4xe^x$

$y' = \dfrac{dy}{dx} = 5\cdot\frac{1}{3}x^{\frac{1}{3}-1}+4\left(1\cdot e^x+x\cdot e^x\right)$

$y' = \dfrac{5}{3\sqrt[3]{x^2}}+4e^x+4xe^x$

(c) $y = \frac{2x-7}{x^2+1}$

$y' = \frac{(2x-7)'(x^2+1) - (2x-7)(x^2+1)^x}{(x^2+1)^2}$

$y' = \frac{2(x^2+1) - (2x-7)2x}{(x^2+1)^2}$

$= \frac{2x^2+2-4x^2+14x}{(x^2+1)^2}$

$y' = \frac{-2x^2+14x+2}{(x^2+1)^2}$

(d) $y = \frac{t^2}{4t^2-3t+2}$

$y' = \frac{(t^2)'(4t^2-3t+2) - t^2(4t^2-3t+2)'}{(4t^2-3t+2)^2}$

$y' = \frac{2t(4t^2-3t+2) - t^2(8t-3)}{(4t^2-3t+2)^2}$

$y' = \frac{8t^3 - 6t^2 + 4t - 8t^3 + 3t^2}{(4t^2-3t+2)^2}$

$y' = \frac{-3t^2+4t}{(4t^2-3t+2)^2}$

(e) $y = \frac{(x-1)(x^2-2x)}{x^4}$

$y = \frac{x^3-3x^2+2x}{x^4}$

$y = x^{-1} - 3x^{-2} + 2x^{-3}$

$y' = -1x^{-2} - 3(-2)x^{-3} + 2(-3)x^{-4}$

$y' = \frac{-1}{x^2} + \frac{6}{x^3} - \frac{6}{x^4}$

or

$y' = \frac{-x^2+6x-6}{x^4}$

(12) $f(t) = 16t^2$

t = time (seconds)

$f(t)$ = distance (feet)

Solution

(a) average speed = $\frac{\text{distance traveled}}{\text{time elapsed}}$

$= \frac{f(4)-f(1)}{4-1} = \frac{16\cdot 4^2 - 16}{3}$

$= \frac{16\cdot 15}{3} = 80$ ft/s

av. speed = 80 ft/s

(b) instantaneous velocity at $t=3$s is $f'(2)$

$f'(t) = 16\cdot 2t = 32t$

$f'(3) = 96$ ft/s

(13) $y = x^2 + x$

let $(a, f(a)) = (a, a^2+a)$ the points of tangency

Let m = slope of the tangent at $x=a$

$$m = \frac{\Delta y}{\Delta x} = \frac{a^2+a+3}{a-2}$$

(2,-3) and (a, a^2+a)

also,

$$m = y'\Big|_{x=a} = 2x+1\Big|_{x=a} = 2a+1$$

$$\Rightarrow \frac{a^2+a+3}{a-2} = 2a+1$$

$$a^2+a+3 = (2a+1)(a-2)$$

$$a^2+a+3 = 2a^2-3a-2$$

$$a^2-4a-5=0$$

$$(a+1)(a-5)=0 \quad \begin{cases} a=-1 \\ a=5 \end{cases}$$

if $a=-1$, $m = 2a+1 = -1$

$m=-1$, (2,-3)

$$y-y_1 = m(x-x_1)$$

$$y+3 = -(x-2)$$

$$y = -x-1$$

if $a=5$, $m = 2a+1 = 11$

$m=11$, (2,-3)

$$y-y_1 = m(x-x_1)$$

$$y+3 = 11(x-2)$$

$$y = 11x -$$

EXTRA CREDIT

(1) $f(x) = \dfrac{x(x^2-1)}{|x^2-1|}$

$$|x^2-1| = \begin{cases} x^2-1 & \text{if } x^2-1 \geq 0 \\ -(x^2-1) & \text{if } x^2-1 < 0 \end{cases}$$

$$|x^2-1| = \begin{cases} x^2-1 & \text{if } x \leq -1 \text{ or } x \geq 1 \\ 1-x^2 & \text{if } -1 < x < 1 \end{cases}$$

so
$$f(x) = \begin{cases} \dfrac{x(x^2-1)}{x^2-1} & \text{if } x<-1 \text{ or } x>1 \\ \dfrac{x(x^2-1)}{1-x^2} & \text{if } -1<x<1 \end{cases}$$

$$f(x) = \begin{cases} x & \text{if } x<-1 \text{ or } x>1 \\ -x & \text{if } -1<x<1 \end{cases}$$

f could be extended to be continuous at $x=1$ or $x=-1$ only if $\lim_{x\to 1} f(x)$ and $\lim_{x\to -1} f(x)$ exist

Note that

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(-x) = -1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x = 1$$

so $\lim_{x\to 1} f(x) = DNE$

so f cannot be extended to be cont. at $x=1$

Similarly for $x=-1$

(2) $P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$

$$\frac{dP}{dV} = nRT\left(\frac{1}{V-nb}\right)' - an^2 (V^{-2})'$$

$$= nRT \cdot \frac{0 - 1 \cdot 1}{(V-nb)^2} - an^2(-2)V^{-3}$$

$$\frac{dP}{dV} = \frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$$

(3) See Textbook or notebook

(4) $u(x) = f(x)\, g(x)$

$v(x) = \frac{f(x)}{g(x)}$

(a)

$$u'(1) = f'(1) g(1) + f(1) g'(1)$$

$$= \frac{2}{1} \cdot 1 + 2 \cdot \left(-\frac{2}{2}\right)$$

$$= 2 - 2 = 0, \text{ so } u'(1) = 0$$

(b) $v'(5) = \frac{f'(5) g(5) - f(5) g'(5)}{g^2(5)}$

$$v'(5) = \frac{-\frac{1}{3} \cdot 2 - 3 \cdot \frac{2}{3}}{2^2} = \frac{-\frac{2}{3} - 2}{4}$$

$$v'(5) = \frac{-\frac{8}{3}}{4} = \frac{-8}{3 \cdot 4} = \frac{-2}{3}$$

$$v'(5) = \frac{-2}{3}$$