Math 180 B Fall 2010 Name:

QUIZ #2 @ 80 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1. Do the following:

a) If $y=\sqrt[4]{x^3}$, find $\dfrac{dy}{dx}$.

b) If $f(x)=\dfrac{(x-1)(x^2-2x)}{x^4}$, find $f'(x)$.

c) If $y=6x^3e^{2x}-\dfrac{1}{x^3}$, find $\dfrac{dy}{dx}$.

d) If $r=\left(\dfrac{\sin\theta}{1-\cos\theta}\right)^2$, find $\dfrac{dr}{d\theta}$.

e) If $y=\cos\left(x^4+e^{-2x}\right)$, find $\dfrac{dy}{dx}$.

f) If $y=\cos(t^2)\sin(2t)$, find $\dfrac{dy}{dt}$.

g) If $f(x)=\tan\left(x^5e^{2x}\right)$, find $f'(x)$.

h) If $y=\left(1+\dfrac{1}{x}\right)^3$, find its second derivative y''.

i) If $y=3^x+\log_4\left(x^2\right)+\ln(5x)$, find $\dfrac{dy}{dx}$.

j) If $r=\cos^{-1}\left(2-\theta^2\right)$, find $\dfrac{dr}{d\theta}$.

k) If $y=\tan^{-1}(\ln x)$, find $\dfrac{dy}{dx}$.

l) If $y=\sin\left(3\cos\left(\dfrac{t}{3}\right)\right)$, find $\dfrac{dy}{dt}$

2. Use implicit differentiation to find dy/dx if

$$x^2y+xy^2=6.$$

3. Use logarithmic differentiation to find dy/dx if

$$y=\frac{(x^3+2)\sqrt{x-2}}{x-1}, \qquad x>1.$$

4. Find $dy/d\theta$ if $y=\theta^{\theta+1}$.

5. Find the linearization of $f(x)=x+\dfrac{2}{x}$ at $x=2$.

6. The position of a particle moving along a coordinate line is $s=\sqrt{1+3t}$, with s in meters and t in seconds. Find the particle's velocity and acceleration at $t=6$ seconds.

Quiz 2B- Solutions

(1) (a) $y = \sqrt[4]{x^3} = x^{3/4}$

$\frac{dy}{dx} = \frac{3}{4} x^{\frac{3}{4}-1} = \frac{3}{4} x^{-\frac{1}{4}} = \boxed{\frac{3}{4\sqrt[4]{x}}}$

(b) $f(x) = \frac{(x-1)(x^2-2x)}{x^4} = \frac{x^3-3x^2+2x}{x^4}$

$f(x) = \frac{1}{x} - \frac{3}{x^2} + \frac{2}{x^3}$

$f(x) = x^{-1} - 3x^{-2} + 2x^{-3}$

$f'(x) = -x^{-2} - 3(-2)x^{-3} + 2(-3)x^{-4}$

$f'(x) = \frac{-1}{x^2} + \frac{6}{x^3} - \frac{6}{x^4}$

$\boxed{f'(x) = \frac{-x^2+6x-6}{x^4}}$

(c) $y = 6x^3e^{2x} - \frac{1}{x^3} = 6x^3e^{2x} - x^{-3}$

$\frac{dy}{dx} = 6\left(3x^2e^{2x} + 6x^3e^{2x}\cdot 2\right) - (-3)x^{-4}$

$\boxed{\frac{dy}{dx} = 18x^2e^{2x} + 72x^3e^{2x} + 3x^{-4}}$

(d) $r = \left(\frac{\sin\theta}{1-\cos\theta}\right)^2$

$\frac{dr}{d\theta} = 2\left(\frac{\sin\theta}{1-\cos\theta}\right)\left(\frac{\sin\theta}{1-\cos\theta}\right)'$

$= \frac{2\sin\theta}{1-\cos\theta} \cdot \frac{\cos\theta(1-\cos\theta) - \sin\theta(\sin\theta)}{(1-\cos\theta)^2}$

$\frac{dr}{d\theta} = \frac{2\sin\theta}{1-\cos\theta} \cdot \frac{\cos\theta - 1}{(1-\cos\theta)^2}$

$\boxed{\frac{dr}{d\theta} = \frac{-2\sin\theta}{(1-\cos\theta)^2}}$

(e) $y = \cos(x^4 + e^{-2x})$

$\frac{dy}{dx} = -\sin(x^4+e^{-2x})\cdot(4x^3 - 2e^{-2x})$

$\boxed{\frac{dy}{dx} = -2(2x^3 - e^{-2x})\sin(x^4+e^{-2x})}$

(f) $y = \cos t^2 \sin(2t)$

$\frac{dy}{dt} = -\sin t^2 \cdot 2t \cdot \sin(2t) + \cos t^2 \cdot \cos(2t)\cdot 2$

$\boxed{\frac{dy}{dt} = -2t\sin t^2 \sin(2t) + 2\cos t^2 \cos(2t)}$

(g) $f(x) = \tan(x^5e^{2x})$

$f'(x) = \sec^2(x^5e^{2x})(5x^4e^{2x} + 2x^5e^{2x})$

$\boxed{f'(x) = x^4e^{2x}(5+2x)\sec^2(x^5e^{2x})}$

(h) $y = \left(1+\frac{1}{x}\right)^3$

$y' = 3\left(1+\frac{1}{x}\right)^2\left(1+\frac{1}{x}\right)'$

$y' = 3\left(1+\frac{1}{x}\right)^2\left(\frac{-1}{x^2}\right)$

$y'' = 3\left(2\left(1+\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)\left(\frac{-1}{x^2}\right) + \left(1+\frac{1}{x}\right)^2(-1\cdot -2)x^{-3}\right)$

$$y'' = 3\left(2\left(1+\frac{1}{x}\right)\frac{1}{x^4} + \frac{2}{x^3}\left(1+\frac{1}{x}\right)^2\right)$$

$$y'' = 6\left(1+\frac{1}{x}\right)\frac{1}{x^3}\left(\frac{1}{x}+\left(1+\frac{1}{x}\right)\right)$$

$$\boxed{y'' = \frac{6}{x^3}\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right) \quad \text{OR} \quad y'' = 6x^{-3} + 18x^{-4} + 12x^{-5}}$$

(i) $y = 3^x + \log_4(x^2) + \ln(5x)$

$$\frac{dy}{dx} = 3^x \ln 3 + \frac{1}{x^2 \ln 4} \cdot 2x + \frac{1}{5x}\cdot 5$$

$$\boxed{\frac{dy}{dx} = 3^x \ln 3 + \frac{2}{x \ln 4} + \frac{1}{x}}$$

(j) $r = \cos^{-1}(2-\theta^2)$

$$\frac{dr}{d\theta} = \frac{-1}{\sqrt{1-(2-\theta^2)^2}}(-2\theta)$$

$$\boxed{\frac{dr}{d\theta} = \frac{2\theta}{\sqrt{1-(2-\theta^2)^2}}}$$

(k) $y = \tan^{-1}(\ln x)$

$$\frac{dy}{dx} = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x} \Rightarrow$$

$$\boxed{\frac{dy}{dx} = \frac{1}{x(1+(\ln x)^2)}}$$

(l) $y = \sin\left(3\cos\frac{t}{3}\right)$

$$\frac{dy}{dt} = \cos\left(3\cos\frac{t}{3}\right)\cdot 3\left(-\sin\frac{t}{3}\right)\cdot\frac{1}{3}$$

$$\boxed{\frac{dy}{dt} = -\cos\left(3\cos\frac{t}{3}\right)\sin\frac{t}{3}}$$

(2) $x^2y + xy^2 = 6 \quad \Big/ \frac{d}{dx}$

$$2xy + x^2y' + y^2 + x\cdot 2y\,y' = 0$$

$$x^2y' + 2xyy' = -2xy - y^2$$

$$y'(x^2+2xy) = -2xy - y^2$$

$$\boxed{y' = \frac{-2xy-y^2}{x^2+2xy}}$$

(3) $y = \dfrac{(x^3+2)\sqrt{x-2}}{x-1}$

$$\ln y = \ln\frac{(x^3+2)\sqrt{x-2}}{x-1}$$

$$\ln y = \ln(x^3+2) + \frac{1}{2}\ln(x-2) - \ln(x-1)$$

$$\frac{1}{y}y' = \frac{3x^2}{x^3+2} + \frac{1}{2(x-2)} - \frac{1}{x-1}$$

$$y' = y\left(\frac{3x^2}{x^3+2} + \frac{1}{2(x-2)} - \frac{1}{x-1}\right)$$

$$\boxed{y' = \frac{(x^3+2)\sqrt{x-2}}{x-1}\left(\frac{3x^2}{x^3+2} + \frac{1}{2(x-2)} - \frac{1}{x-1}\right)}$$

(4) $y = \theta^{\theta+1}$

$\ln y = \ln(\theta^{\theta+1})$

$\ln y = (\theta+1)\ln\theta$

$\frac{1}{y}\cdot y' = 1\cdot\ln\theta + (\theta+1)\frac{1}{\theta}$

$y' = y\left(\ln\theta + \frac{\theta+1}{\theta}\right)$

$$\boxed{y' = \theta^{\theta+1}\left(\ln\theta + \frac{\theta+1}{\theta}\right)}$$

(5) $f(x) = x + \frac{2}{x}$

We want to find the equation of the tangent line at $(2,3)$.

First, we find $m = f'(2)$

$f'(x) = 1 - \frac{2}{x^2}$

$f'(2) = 1 - \frac{2}{4} = \frac{1}{2}$ so $m = \frac{1}{2}$

$y - 3 = \frac{1}{2}(x-2)$

$y = \frac{1}{2}x + 2$

so The tangent line is

$L(x) = \frac{1}{2}x + 2$ and

$f(x) \approx L(x)$

$$\boxed{x + \frac{2}{x} \approx \frac{x}{2} + 2 \text{ when } x \text{ is near } 2}$$

(6) $s = \sqrt{1+3t}$

s = position fct. (m)

t = time (s)

let $v(t)$ = velocity fct.

$v(t) = \frac{ds}{dt} = \frac{1}{2\sqrt{1+3t}}\cdot 3$

$v(t) = \frac{3}{2\sqrt{1+3t}}$

Then, $v(6) = \frac{3}{2\sqrt{19}}\ \frac{m}{s}$

$$\boxed{v(6) = \frac{3\sqrt{19}}{38}\ \frac{m}{s}}$$

let $a(t)$ = acceleration fct.

$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{3}{2\sqrt{1+3t}}\right)$

$= \frac{d}{dt}\left(\frac{3}{2}(1+3t)^{-\frac{1}{2}}\right)$

$= \frac{3}{2}\cdot\frac{-1}{2}(1+3t)^{-\frac{3}{2}}\cdot 3$

$= \frac{-9}{4(1+3t)\sqrt{1+3t}}$

$a(t) = \frac{-9}{4(1+3t)\sqrt{1+3t}}$

Then, $a(6) = \frac{-9}{4(19)\sqrt{19}}\ \frac{m}{s^2}$

$$\boxed{a(6) = \frac{-9\sqrt{19}}{4\cdot 19^2}\ \frac{m}{s^2}}$$