Math 130 Winter 2010 Name:

QUIZ #3 @ 100 points

Write neatly. Show all work. **Write all responses on separate paper.** Clearly label the exercises.

1) Solve the following system using matrices: $\begin{cases} 2x - y + 3z = 0 \\ x + 2y - z = 5 \\ 2y + z = 1 \end{cases}$

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2) Solve the following system using matrices: $\begin{cases} x + 3y - 2z - w = 9 \\ 4x + y + z + 2w = 2 \\ -3x - y + z - w = -5 \\ x - y - 3z - 2w = 2 \end{cases}$

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3) Let $A = \begin{pmatrix} -3 & 1 & 1 \\ 1 & -1 & 2 \\ 5 & 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

Do the following operations. If not defined, say so and explain why.

a) $2A + B$ b) AB c) B^2

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4) Find and simplify:

a) $\displaystyle\sum_{i=1}^{6} \binom{6}{i}$ b) $\displaystyle\sum_{n=1}^{4} \frac{(n+2)!}{(n-1)!}$

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5) Use the Binomial Theorem to expand . Show all work. Do not just write an answer.

a) $(2x+3y)^5$ b) $\left(\frac{1}{2}a - \frac{1}{3}b\right)^4$

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6) Answer all questions for each sequence:

a) $1, \frac{1}{3}, \frac{1}{9}, \ldots$ b) -3, -7, -11, …

i)What type of sequence is it?
ii) Specify the first term and the common difference or ratio.
iii) Find a formula for the general term a_n and simplify.
iv) Find a formula for the sum of the first n terms and simplify.
v) What is the sum of the first 100 terms?
vi) Find the sum of all the terms (infinite sum) when appropriate. Explain why you can or cannot find the sum (in other words, why the sequence is convergent or not).

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7) Find a fraction representation for the rational number $2.\overline{13}$. Show all work.

EXTRA CREDIT

#1 @ 3 points

Let $\log 2, \log 4, \log 8, \ldots$ be a sequence. Answer the following:

a)What type of sequence is it?

b) Specify the first term and the common difference or ratio.

c) Find a formula for the general term a_n and simplify.

d) What is the sum of the first 100 terms?

#2 @ 3 points

You have 1 penny, 1 nickel, 1 dime, and 1 quarter. How many different sums of money can you form using one or more of these coins? Show a mathematical solution.

#3 @ 4 points

Let $\left(x+y^2\right)^{30}$. Find the following:

a) Find the 17$^{\text{th}}$ term . Show all work.

b) Find the coefficient of the term that contains x^{11}. Show all work.

(7) $2.\overline{13} = 2.131313\ldots$

$$= 2 + \underbrace{\frac{13}{100} + \frac{13}{10{,}000} + \frac{13}{1{,}000{,}000}}_{} + \cdots$$

geometric series with

$a_1 = \frac{13}{100}$ and $r = \frac{1}{100}$

Because $|r| < 1$, $S_\infty = \frac{a_1}{1-r}$

$$2.\overline{13} = 2 + \frac{\frac{13}{100}}{1 - \frac{1}{100}}$$

$$= 2 + \frac{\frac{13}{100}}{\frac{99}{100}}$$

$$= 2 + \frac{13}{99}$$

$$\boxed{2.\overline{13} = \frac{211}{99}}$$

MATH 130 Quiz 3 - Solutions

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$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ \boxed{4} & 1 & 1 & 2 & 2 \\ \boxed{-3} & -1 & 1 & -1 & -5 \\ \boxed{1} & -1 & -3 & -2 & 2 \end{array}\right] \xrightarrow[\substack{R_3 \to 3R_1 + R_3 \\ R_4 \to -R_1 + R_4}]{R_2 \to -4R_1 + R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & \boxed{8} & -5 & -4 & 22 \\ 0 & -4 & -1 & -1 & -7 \end{array}\right] \xrightarrow[R_3 \to 2R_4 + R_3]{}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -7 & -6 & 8 \\ 0 & -4 & -1 & -1 & -7 \end{array}\right] \xrightarrow[R_4 \to -11R_4]{R_2 \to 4R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -44 & 36 & 24 & -136 \\ 0 & 0 & -7 & -6 & 8 \\ 0 & \boxed{44} & 11 & 11 & 77 \end{array}\right] \xrightarrow[R_4 \to R_2 + R_4]{R_2 \to \frac{1}{4}R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -7 & -6 & 8 \\ 0 & 0 & 47 & 35 & -59 \end{array}\right] \xrightarrow[R_4 \to 7R_4]{R_3 \to 47R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 9 \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -329 & -282 & 376 \\ 0 & 0 & \boxed{329} & 245 & -413 \end{array}\right] \xrightarrow[R_4 \to R_3 + R_4]{R_3 \to \frac{1}{47}R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & -1 & \\ 0 & -11 & 9 & 6 & -34 \\ 0 & 0 & -7 & -6 & 8 \\ 0 & 0 & 0 & -37 & -3 \end{array}\right]$$

4th row: $-37w = -37 \Rightarrow$

3rd row: $-7z - 6w = 8$

$-7z - 6 = 8$

$-7z = 14 \Rightarrow$

2nd row: $-11y + 9z$

$-11y - 18 +$

$-11y - 12 = -$

$11y =$

1st row $x + 3y - 2z -$

$x + 6 + 4 -$

$x + 9 =$

The solution is $(0,$

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$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 1 & 2 & -1 & 5 \\ 0 & 2 & 1 & 1 \end{array}\right] \xrightarrow{R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ \boxed{2} & -1 & 3 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right] \xrightarrow[R_2 \to -2R_1 + R_2]{}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -5 & 5 & -10 \\ 0 & 2 & 1 & 1 \end{array}\right] \xrightarrow[R_2 \to]{}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -1 & 1 & -2 \\ 0 & \boxed{2} & 1 & 1 \end{array}\right] \xrightarrow[R_3 \to 2R_2 + R_3]{}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 3 & -3 \end{array}\right]$$

3rd row: $3z = -3 \Rightarrow z = -1$

2nd row: $-y + z = -2$

$-y - 1 = -2 \Rightarrow y = 1$

1st row. $x + 2y - z = 5$

$x + 2 + 1 = 5 \Rightarrow x = 2$

The solution is $(2, 1, -1)$

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(3) (a) $2A + B =$

$$2\begin{pmatrix} -3 & 1 & 1 \\ 1 & -1 & 2 \\ 5 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} =$$

$$\begin{pmatrix} -6 & 2 & 2 \\ 2 & -2 & 4 \\ 10 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} =$$

$$\begin{pmatrix} -5 & 8 & 3 \\ 4 & -2 & 5 \\ 10 & 1 & -1 \end{pmatrix}$$

(b) $AB =$

$$\begin{pmatrix} -3 & 1 & 1 \\ 1 & -1 & 2 \\ 5 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} =$$

$$\begin{pmatrix} -1 & -17 & -3 \\ -1 & 8 & -2 \\ 5 & 30 & 5 \end{pmatrix}$$

(c) $B^2 =$

$$\begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 6 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} =$$

$$\begin{pmatrix} 13 & 7 & 6 \\ 2 & 13 & 1 \\ 2 & -1 & 2 \end{pmatrix}$$

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(4) (a) $\sum_{i=1}^{6} \binom{6}{i} = \binom{6}{1} + \binom{6}{2} + \binom{6}{3}$

$+ \binom{6}{4} + \binom{6}{5} + \binom{6}{6}$

$= \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{2} + \binom{6}{1} + ($

$= 6 + \frac{{}^{3}\cancel{6}\cdot 5}{1\cdot\cancel{2}} + \frac{\cancel{6}\cdot 5\cdot 4}{1\cdot\cancel{2}\cdot\cancel{3}} + 15 + 6 +$

$= 6 + 15 + 20 + 15 + 6 + 1 = \boxed{6}$

(b) $\sum_{n=1}^{4} \frac{(n+2)!}{(n-1)!} =$

$= \sum_{n=1}^{4} \frac{(n-1)!\, n(n+1)(n+2)}{(n-1)!}$

$= \sum_{n=1}^{4} n(n+1)(n+2)$

$= 1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + 3\cdot 4\cdot 5 + 4$

$= 6 + 24 + 60 + 120$

$= \boxed{210}$

(5) (a) $(2x+3y)^5 =$

$= \binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4(3y) + \binom{5}{2}(2x)^3(3y)^2$
$+ \binom{5}{3}(2x)^2(3y)^3 + \binom{5}{4}(2x)(3y)^4 + \binom{5}{5}(3y)^5$

$= 1\cdot 32x^5 + 5\cdot 16x^4\cdot 3y + \frac{5\cdot 4}{1\cdot 2}8x^3\cdot 9y^2$
$+ 10\cdot 4x^2\cdot 27y^3 + 5\cdot 2x\cdot 81y^4 + 1\cdot 243y^5$

$= 32x^5 + 240x^4y + 720x^3y^2 +$
$+ 1080x^2y^3 + 810xy^4 + 243y^5$

(b) $\left(\frac{1}{2}a - \frac{1}{3}b\right)^4 =$

$= \binom{4}{0}\left(\frac{1}{2}a\right)^4 - \binom{4}{1}\left(\frac{1}{2}a\right)^3\left(\frac{1}{3}b\right) + \binom{4}{2}\left(\frac{1}{2}a\right)^2\left(\frac{1}{3}b\right)^2$
$- \binom{4}{3}\left(\frac{1}{2}a\right)\left(\frac{1}{3}b\right)^3 + \binom{4}{4}\left(\frac{1}{3}b\right)^4$

$= 1\cdot\frac{1}{16}a^4 - 4\cdot\frac{1}{8}a^3\cdot\frac{1}{3}b + \frac{4\cdot 3}{1\cdot 2}\frac{1}{4}a^2\frac{1}{9}b^2$
$- 4\cdot\frac{1}{2}a\cdot\frac{1}{27}b^3 + 1\cdot\frac{1}{81}b^4$

$= \frac{1}{16}a^4 - \frac{1}{6}a^3b + \frac{1}{6}a^2b^2 -$
$- \frac{2}{27}ab^3 + \frac{1}{81}b^4$

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(6) (a) $1, \frac{1}{3}, \frac{1}{9}, \ldots$

$a_1 = 1, \; a_2 = \frac{1}{3}, \; a_3 = \frac{1}{9}$

$\frac{a_2}{a_1} = \frac{1}{3}$; $\frac{a_3}{a_2} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$

Therefore, this is a geometric sequence with $\boxed{a_1 = 1 \text{ and } r = \frac{1}{3}}$
(i)(ii)

(iii) $a_n = a_1 r^{n-1}$
$\boxed{a_n = \left(\frac{1}{3}\right)^{n-1}}$

(iv) $S_n = \frac{a_1(1-r^n)}{1-r}$

$S_n = \frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}$; $\boxed{S_n = \frac{3}{2}\left(1-\frac{1}{3^n}\right)}$

(v) $\boxed{S_{100} = \frac{3}{2}\left(1-\frac{1}{3^{100}}\right)}$

(vi) We have a geometric sequence with $r = \frac{1}{3}$, $|r|<1$

so $S_\infty =$

$S_\infty = \frac{1}{1-\frac{1}{3}}$; $\boxed{S_\infty = \frac{3}{2}}$

(b) $-3, -7, -11, \ldots$

$a_1 = -3, \; a_2 = -7, \; a$

$a_2 - a_1 = -4$; $a_3 - a_2 = -4$

Therefore, this is an arithmetic sequence with $\boxed{\begin{matrix} a_1 = -3 \\ d = -4 \end{matrix}}$
(i), (ii)

(iii) $a_n = a_1 + (n-1)d$
$a_n = -3 + (n-1)(-4)$
$\boxed{a_n = 1-4n}$

(iv) $S_n = \frac{n(a_1+a_n)}{2} = \frac{n(-3+1-4n)}{2}$
$\boxed{S_n = -n(1+2n)}$

(v) $S_{100} = -100(1+200)$
$\boxed{S_{100} = -20{,}100}$

(vi) S_∞ - not defined for arith.