Quiz #2 - SOLUTIONS

① $f(x) = -4x^3 - 13x^2 - 4x + 12$

ⓐ
$$\begin{array}{r|rrrr} & -4 & -13 & -4 & 12 \\ \hline 3 & -4 & -25 & -79 & -225 \end{array} \; R$$

$f(x) = (x-3)(-4x^2 - 25x - 79) - 225$

ⓑ max. number of real zeros is 3.

ⓒ $f(x)$ has one variation in sign, therefore there is 1 positive real zero.

$f(-x) = -4(-x)^3 - 13(-x)^2 - 4(-x) + 12$

$f(-x) = \underbrace{4x^3 - 13x^2}_{1} \underbrace{+ 4x}_{2} + 12$

$f(-x)$ has two variations in sign so there could be 2 or 0 negative zeros

ⓓ The Rational Zeros Theorem can be applied because all the coefficients are rational numbers and the constant term is nonzero

$\frac{p}{q} = \frac{\text{factor of } 12}{\text{factor of } (-4)}$

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 4}$

$\frac{p}{q} \in \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}\}$

ⓔ note that $f(1) \neq 0$

$$\begin{array}{r|rrrr} & -4 & -13 & -4 & 12 \\ \hline -2 & -4 & -5 & 6 & 0 \end{array} \; R$$

$f(x) = (x+2)(-4x^2 - 5x + 6)$

$$\begin{array}{r|rrr} & -4 & -5 & 6 \\ \hline -2 & -4 & 3 & 0 \end{array} \; R$$

$f(x) = (x+2)(x+2)(-4x+3)$

$f(x) = (x+2)^2(-4x+3)$

The zeros are:

$\begin{cases} x = -2, \text{ multiplicity } 2 \\ x = \frac{3}{4}, \text{ multiplicity } 1 \end{cases}$

ⓕ $f(x) = (x+2)^2(-4x+3)$

ⓖ The end behavior is given by the leading term $-4x^3$

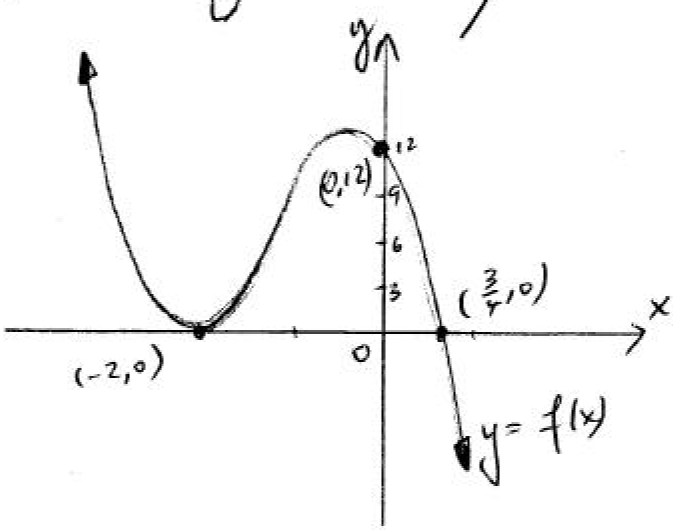
when $x \to \infty$, $y \to -\infty$

when $x \to -\infty$, $y \to \infty$

ⓗ x-int: $(-2, 0)$ and $(\frac{3}{4}, 0)$

y-int: $(0, 12)$

x	$-\infty$	-2	0	$\frac{3}{4}$	∞
$f(x)$	∞	0	12	0	$-\infty$
	$\nwarrow$	$m=2$		$m=1$	$\searrow$



(2) $x = 1-\sqrt{2} \Rightarrow x = 1+\sqrt{2}$

$x = 3+i \Rightarrow x = 3-i$

$x = -5$

$x = \frac{2}{3}$

$$f(x) = a(x-(1-\sqrt{2}))(x-(1+\sqrt{2}))(x-(3+i))(x-(3-i))(x-(-5))(x-\tfrac{2}{3})$$

let $a=1$

$$f(x) = (x-1+\sqrt{2})(x-1-\sqrt{2})(x-3-i)(x-3+i)(x+5)(x-\tfrac{2}{3})$$

$$f(x) = ((x-1)^2-(\sqrt{2})^2)((x-3)^2-i^2)(x+5)(x-\tfrac{2}{3})$$

$$f(x) = (x^2-2x+1-2)(x^2-6x+9-(-1))(x+5)(x-\tfrac{2}{3})$$

$$f(x) = (x^2-2x-1)(x^2-6x+10)(x+5)(x-\tfrac{2}{3})$$

(3) $f(x) = x^4-6x^3+22x^2-30x+13$

The Rational Zeros Theorem can be applied (all coefficients are rational and the constant $\neq 0$) so the list of possible rational zeros is:

$$\frac{p}{q} = \frac{\text{factors of } 13}{\text{factors of } 1} = \frac{\pm 1, \pm 13}{\pm 1}$$

$$\frac{p}{q} \in \{\pm 1, \pm 13\}$$

	1	−6	22	−30	13
1	1	−5	17	−13	0 R

$$f(x) = (x-1)(x^3-5x^2+17x-13)$$

	1	−5	17	−13
1	1	−4	13	0 R

$$f(x) = (x-1)(x-1)(x^2-4x+13)$$

$$f(x) = (x-1)^2(x^2-4x+13)$$

Zeros:

$x=1$, multiplicity 2

and

$x^2-4x+13=0$

$$x = \frac{4 \pm \sqrt{16-4(13)}}{2}$$

$$x = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2}$$

$x = 2 \pm 3i$, each of multiplicity 1

(4) $f(x) = \dfrac{4x^2+4x-24}{x^2-3x-10}$

(a) $f(x) = \dfrac{4(x^2+x-6)}{(x-5)(x+2)}$

$f(x) = \dfrac{4(x+3)(x-2)}{(x-5)(x+2)}$

(b) $x \in \mathbb{R} \setminus \{5, -2\}$

(c) V.A. $x=5,\ x=-2$

H.A. $y=4$

(d) $f(x) \overset{?}{=} 4$

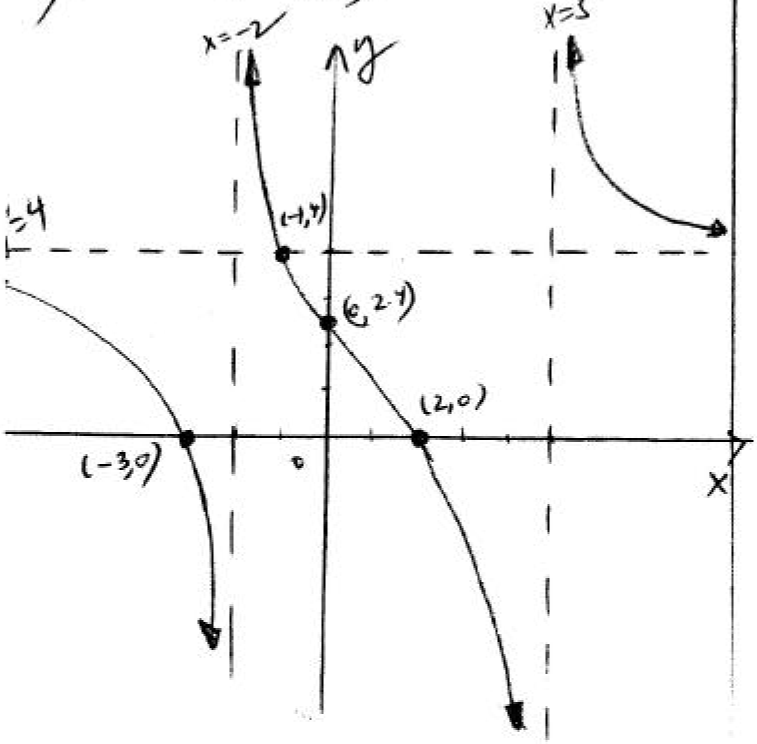
$\dfrac{4x^2+4x-24}{x^2-3x-10} = 4$ iff

$\cancel{4x^2}+4x-24 = \cancel{4x^2}-12x-40$

$4x+12x = -40+24$

$16x = -16 \Rightarrow x=-1$

so there is one intersection point $(-1,4)$



(e) for the graph we also need intercepts

x-n: $y=0$ iff $x=-3,\ x=2$

$(-3,0)$ and $(2,0)$

y-n: $x=0,\ y=\dfrac{-24}{-10}=2.4$

$(0, 2.4)$

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(5) (a) $f(x) = \dfrac{x^2-1}{x+3}$

Domain: $x \neq -3$

$$\begin{cases} \text{V.A. } x=-3 \\ \text{H.A. none} \\ \text{O.A. } y=x-3 \end{cases}$$

$$\begin{array}{r|rrr} & 1 & 0 & -1 \\ \hline -3 & 1 & -3 & (8)\ R \end{array}$$

(b) $g(x) = \dfrac{4-3x}{2x+1}$

$g(x) = \dfrac{-3x+4}{2x+1}$

Domain: $x \neq -\frac{1}{2}$

$$\begin{cases} \text{V.A. } x=\frac{-1}{2} \\ \text{H.A. } y=\frac{-3}{2} \\ \text{O.A. none} \end{cases}$$