

TEST #2 @ 165 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

PART I – Solve all problems:

1. Simplify the following expression. Write the final answer using only positive exponents (if necessary).

$$\left(\frac{mn^{-2}p^3}{m^{-1}np^{-4}} \right)^{-2} \left(\frac{m^3np^{-2}}{m^{-2}n^2p^3} \right)^{-1}$$

2. Let $P(x) = 2x^5 + 3x^4 - \frac{1}{2}x^3 - 10x - 2$ be a polynomial in x . Answer the following questions:

- How many terms does it have?
- What is the degree of the polynomial?
- What is the constant term?
- Find $P(0)$, $P(-1)$, and $P(a^2)$ and simplify.

3. Use long division to find:

$$\frac{2x^3 - 5x^2 - 6x + 15}{x - 3}$$

4. Factor each expression as completely as possible. If prime, state so.

- | | |
|-----------------------------------|----------------------|
| a) $26a^4b - 34a^3c^2 + 28a^2b^3$ | h) $1 - 16m^4$ |
| b) $49 - m^2$ | i) $9y^2 + 12y - 15$ |
| c) $a^2 - 4a - ab + 4b$ | j) $3t^2 - 15t + 16$ |
| d) $64x^3 - 27$ | k) $c^2 + 16$ |
| e) $25x^2 + 100$ | l) $3a^2 + 24a + 48$ |
| f) $2x + x^2 - 15$ | m) $8 + n^3$ |
| g) $3x^2 - 11x - 20$ | n) $6a^2 + 40a + 24$ |

5. Do the following operations:

a) $3^{-2} + \left(\frac{1}{2}\right)^{-1} + (-2)^{-1} + 5^0$

b) $(x^3 + 4)^2$

c) $(3a - 2c)^2$

d) $\frac{x+5}{x+10} \div \left(\frac{x^2+10x+25}{x^2+10x} \cdot \frac{10x}{x^2+15x+50} \right)$

e) $\frac{\frac{1}{x^2} + \frac{1}{y^2}}{\frac{1}{x} - \frac{1}{y}}$

f) $\frac{8}{a^2+6a} - \frac{3}{a^2+4a-12}$

g) $\frac{4y}{y^2-1} - \frac{5}{y^2+2y+1}$

h) $\frac{m}{m^2-1} + \frac{m-1}{m^2+2m+1}$

6. Solve the following equations by factoring.

a) $x(2x-3) = -1$

c) $(a-2)(a-4) = 15$

b) $3m^2 + 6m = 0$

d) $t^3 + 35t = 12t^2$

7. Solve each equation. Write conditions whenever necessary.

a) $5x \left(x - \frac{3}{2} \right) \left(5x - \frac{2}{3} \right) \left(3 - \frac{x}{4} \right) = 0$

d) $\frac{1}{x-1} + \frac{1}{2} = \frac{2}{x^2-1}$

b) $\frac{3x-1}{x-2} = \frac{5}{x-2} + 1$

e) $I = \frac{E}{R+r}$ solve for r.

c) $\frac{x+4}{x^2-3x+2} - \frac{5}{x^2-4x+3} = \frac{x-4}{x^2-5x+6}$

f) $m = \frac{Ry}{t}$ solve for t.

8. A stone is thrown upward from the top of a building. The height of the stone above the ground t seconds later is given by the formula

$$h = -16t^2 + 32t + 240$$

where h is in feet.

- Find the initial height of the stone.
- Find the height of the stone after 1 second.
- When will the stone be 256 feet above the ground?
- How long will it take to hit the ground?

PART II of the test on next page.

PART II – Choose TWO problems. You can solve one more for extra credit.

1. Hooke's law for an elastic spring states that the distance a spring stretches varies directly with the force applied. If a force of 75 lb stretches a certain spring 15 inches, how much will a force of 150 lb stretch the spring?
2. A ladder is leaning against a building. The distance from the bottom of the ladder to the building is 4 ft less than the length of the ladder. How high up the side of the building is the top of the ladder if that distance is 2 ft less than the length of the ladder?
3. A boat goes 7 mph in still water. It takes as long to go 20 mi upstream as 50 mi downstream. Find the speed of the current.
4. Ella can paint a room, working alone, in 5 hours. Frank can do the job in 4 hours. How long will it take them to paint the room if they work together?

TEST 2 - SOLUTIONS

$$\textcircled{1} \left(\frac{mn^{-2}p^3}{m^{-1}n^4p^{-4}} \right)^{-2} \left(\frac{m^3n^2p^{-2}}{m^{-2}n^2p^3} \right)^{-1} =$$

$$\left(m^{1+1} n^{-2-4} p^{3+4} \right)^{-2} \left(m^{3+2} n^{1-2} p^{-2-3} \right)^{-1} =$$

$$\left(m^2 n^{-3} p^7 \right)^{-2} \left(m^5 n^{-1} p^{-5} \right)^{-1} =$$

$$m^{-4} n^6 p^{-14} m^{-5} n p^5 =$$

$$m^{-4-5} n^{6+1} p^{-14+5} =$$

$$m^{-9} n^7 p^{-9} = \left[\frac{n^7}{m^9 p^9} \right]$$

$$\textcircled{2} P(x) = 2x^5 + 3x^4 - \frac{1}{2}x^3 - 10x - 2$$

a) 5 terms

b) degree 5

c) -2

$$d) \boxed{P(0) = -2}$$

$$P(-1) = 2(-1) + 3(1) - \frac{1}{2}(-1) - 10(-1) - 2$$

$$P(-1) = -2 + 3 + \frac{1}{2} + 10 - 2$$

$$= 9 + \frac{1}{2} = \frac{19}{2}$$

$$\boxed{P(-1) = \frac{19}{2}}$$

$$P(a^2) = 2(a^2)^5 + 3(a^2)^4 - \frac{1}{2}(a^2)^3 - 10(a^2) - 2$$

$$\boxed{P(a^2) = 2a^{10} + 3a^8 - \frac{1}{2}a^6 - 10a^2 - 2}$$

$$\textcircled{3} \begin{array}{r} 2x^2 + x - 3 \\ x-3 \overline{) 2x^3 - 5x^2 - 6x + 15} \\ \underline{-2x^3 + 6x^2} \\ 1 x^2 - 6x + 15 \\ \underline{-x^2 + 3x} \\ 1 -3x + 15 \\ \underline{+ 3x - 9} \\ 6 \end{array}$$

Therefore,

$$\boxed{\frac{2x^3 - 5x^2 - 6x + 15}{x-3} = 2x^2 + x - 3 + \frac{6}{x-3}}$$

(4)

$$a) 26a^4b - 34a^3c^2 + 28a^2b^3 =$$

$$\boxed{2a^2(13a^2b - 17ac^2 + 14b^3)}$$

$$b) 49 - m^2 = 7^2 - m^2 = \boxed{(7-m)(7+m)}$$

$$c) a^2 - 4a - ab + 4b =$$

$$a(a-4) - b(a-4) =$$

$$\boxed{(a-4)(a-b)}$$

$$d) 64x^3 - 27 = (4x)^3 - 3^3 = (4x-3)((4x)^2 + 4x(3) + 3^2) = \boxed{(4x-3)(16x^2 + 12x + 9)}$$

$$e) 25x^2 + 100 =$$

$$\boxed{25(x^2 + 4)}$$

$$f) 2x + x^2 - 15 = x^2 + 2x - 15 =$$

$$\boxed{(x+5)(x-3)}$$

$$\begin{array}{r} p = -5 < -3 \\ s = 2 \\ 15 = 5 \cdot 3 \end{array}$$

$$g) 3x^2 - 11x - 20 =$$

$$3x^2 + 4x - 15x - 20 =$$

$$x(3x+4) - 5(3x+4) =$$

$$\boxed{(3x+4)(x-5)}$$

$$\begin{array}{r} p = -60 < -15 \\ s = -11 \\ 60 = 15 \cdot 4 \end{array}$$

-2-

$$\begin{aligned}
 h) \quad 1-16m^4 &= 1^2 - (4m^2)^2 \\
 &= (1-4m^2)(1+4m^2) \\
 &= (1^2 - (2m)^2)(1+4m^2) \\
 &= \boxed{(1-2m)(1+2m)(1+4m^2)}
 \end{aligned}$$

$$\begin{aligned}
 l) \quad 9y^2 + 12y - 15 &= \\
 \boxed{3(3y^2 + 4y - 5)} & \quad \begin{aligned} p &= -15 < - \\ s &= 4 \\ 15 &= 5 \cdot 3 \\ &= 15 \cdot 1 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 j) \quad \boxed{3t^2 - 15t + 16} &= \text{prime} \\
 p &= 48 < - \\
 s &= -15 \\
 48 &= 2 \cdot 24 \\
 &= 4 \cdot 12 \\
 &= 8 \cdot 6 \\
 &= 16 \cdot 3 \\
 &= 48 \cdot 1
 \end{aligned}$$

$$k) \quad \boxed{c^2 + 16} = \text{prime}$$

$$\begin{aligned}
 e) \quad 3a^2 + 24a + 48 &= \\
 3(a^2 + 8a + 16) &= \\
 \boxed{3(a+4)^2}
 \end{aligned}$$

$$\begin{aligned}
 m) \quad 8 + n^3 &= 2^3 + n^3 \\
 &= (2+n)(2^2 - 2n + n^2) \\
 &= \boxed{(2+n)(4 - 2n + n^2)}
 \end{aligned}$$

$$\begin{aligned}
 n) \quad 6a^2 + 40a + 24 &= \\
 2(3a^2 + 20a + 12) &= \\
 2(3a^2 + 18a + 2a + 12) &= \\
 2(3a(a+6) + 2(a+6)) &= \\
 \boxed{2(a+6)(3a+2)} & \quad \begin{aligned} p &= 36 < +18 \\ s &= 2 \\ 36 &= 18 \cdot 2 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad a) \quad 3^{-2} + \left(\frac{1}{2}\right)^{-1} + (-2)^{-1} + 5^0 &= \\
 \frac{1}{3^2} + \frac{1}{\left(\frac{1}{2}\right)^1} + \frac{1}{(-2)^1} + 1 &= \\
 \frac{2^1}{1} + \frac{18^1}{1} - \frac{9^1}{2} + \frac{18^1}{1} &=
 \end{aligned}$$

$$\begin{aligned}
 LCO &= 18 \\
 \frac{2 + 36 - 9 + 18}{18} &= \boxed{\frac{47}{18}}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad (x^3 + 4)^2 &= (x^3)^2 + 2(x^3)4 + 4^2 \\
 &= \boxed{x^6 + 8x^3 + 16}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad (3a - 2c)^2 &= (3a)^2 - 2(3a)(2c) + (2c)^2 \\
 &= \boxed{9a^2 - 12ac + 4c^2}
 \end{aligned}$$

$$d) \quad \frac{x+5}{x+10} \div \left(\frac{x^2 + 10x + 25}{x^2 + 10x} \cdot \frac{10x}{x^2 + 15x + 50} \right) =$$

$$\frac{x+5}{x+10} \div \left(\frac{(x+5)^2}{x(x+10)} \cdot \frac{10x}{(x+10)(x+5)} \right) =$$

$$\frac{x+5}{x+10} \div \frac{10(x+5)}{(x+10)^2} =$$

$$\frac{x+5}{x+10} \cdot \frac{(x+10)^2}{10(x+5)} = \boxed{\frac{x+10}{10}}$$

$$\begin{aligned}
 c) \quad \frac{\frac{1^2}{x^2} + \frac{x^2}{y^2}}{\frac{y^2 + x^2}{x^2 y^2}} &= \frac{y^2 + x^2}{x^2 y^2} \\
 \frac{\frac{y^2}{x} - \frac{x^2}{y}}{x^2 y^2} &= \frac{y-x}{xy} \\
 &= \frac{x^2 - y^2}{x^2 y^2} \cdot \frac{xy}{y-x} \\
 &= \boxed{\frac{x^2 + y^2}{x^2 y^2 (y-x)}}
 \end{aligned}$$

$$f) \frac{8}{a^2+6a} - \frac{3}{a^2+4a-12} =$$

$$\frac{a^2}{8} - \frac{a^2}{3}$$

$$\frac{8}{a(a+6)} - \frac{3}{(a+6)(a-2)} =$$

$$\text{LCD} = a(a+6)(a-2)$$

$$\frac{8(a-2) - 3a}{a(a+6)(a-2)} =$$

$$\frac{8a-16-3a}{a(a+6)(a-2)} = \boxed{\frac{5a-16}{a(a+6)(a-2)}}$$

$$g) \frac{4y}{y^2-1} - \frac{5}{y^2+2y+1} =$$

$$\frac{y^2}{4y} - \frac{y^2}{5}$$

$$\frac{4y}{(y-1)(y+1)} - \frac{5}{(y+1)^2} =$$

$$\text{LCD} = (y+1)^2(y-1)$$

$$\frac{4y(y+1) - 5(y-1)}{(y-1)(y+1)^2} =$$

$$\frac{4y^2+4y-5y+5}{(y-1)(y+1)^2} = \boxed{\frac{4y^2-y+5}{(y-1)(y+1)^2}}$$

$$h) \frac{m}{m^2-1} + \frac{m-1}{m^2+2m+1} =$$

$$\frac{m+1}{m} + \frac{m-1}{m-1}$$

$$\frac{m+1}{(m+1)(m-1)} + \frac{m-1}{(m+1)^2} =$$

$$\text{LCD} = (m-1)(m+1)^2$$

$$\frac{m(m+1) + (m-1)^2}{(m-1)(m+1)^2} =$$

$$\frac{m^2+m+m^2-2m+1}{(m-1)(m+1)^2} =$$

$$\boxed{\frac{2m^2-m+1}{(m-1)(m+1)^2}}$$

c) a) $x(2x-3) = -1$
 $2x^2 - 3x + 1 = 0$

$p = 2$ $q = -3$ $r = -1$
 $s = -3$
 $2 = 1 \cdot 2$

$$2x^2 - x - 2x + 1 = 0$$

$$x(2x-1) - (2x-1) = 0$$

$$(2x-1)(x-1) = 0$$

$$2x-1=0 \text{ OR } x-1=0$$

$$x = \frac{1}{2}$$

$$x = 1$$

$$\boxed{x \in \left\{ \frac{1}{2}, 1 \right\}}$$

b) $3m^2 + 6m = 0$
 $3m(m+2) = 0$
 $m = 0 \text{ OR } m+2 = 0$
 $m = -2$

$$\boxed{m \in \{0, -2\}}$$

c) $(a-2)(a-4) = 15$
 $a^2 - 6a + 8 - 15 = 0$
 $a^2 - 6a - 7 = 0$
 $(a-7)(a+1) = 0$
 $a-7=0 \text{ OR } a+1=0$
 $a = 7 \quad a = -1$

$$\boxed{a \in \{7, -1\}}$$

-4-

$$d) t^3 + 35t = 12t^2$$

$$t^3 - 12t^2 + 35t = 0$$

$$t(t^2 - 12t + 35) = 0$$

$$t(t-7)(t-5) = 0$$

$$t=0 \text{ OR } t-7=0 \text{ OR } t-5=0$$

$$t=7 \qquad \qquad \qquad t=5$$

$$\boxed{t \in \{0, 7, 5\}}$$

$$2x = 4$$

$$x = 2 \text{ not possible}$$

Therefore, no solutions

$$\boxed{x \in \emptyset}$$

(7) a) $5x(x - \frac{3}{2})(5x - \frac{2}{3})(3 - \frac{x}{4}) = 0$

$$x = 0 \text{ OR}$$

$$x - \frac{3}{2} = 0 \Rightarrow x = \frac{3}{2} \text{ OR}$$

$$5x - \frac{2}{3} = 0 \Rightarrow 5x = \frac{2}{3} \Rightarrow x = \frac{2}{15} \text{ OR}$$

$$3 - \frac{x}{4} = 0 \Rightarrow 3 = \frac{x}{4} \Rightarrow x = 12$$

$$\boxed{x \in \{0, \frac{3}{2}, \frac{2}{15}, 12\}}$$

c) $\frac{x+4}{x^2-3x+2} - \frac{5}{x^2-4x+3} = \frac{x-4}{x^2-5x+6}$

$$\frac{x-3}{x+4} - \frac{x-2}{5} = \frac{x-4}{(x-2)(x-3)}$$

Conditions: $\begin{cases} x \neq 1 \\ x \neq 2 \\ x \neq 3 \end{cases}$

$$LCO = (x-1)(x-2)(x-3)$$

$$(x-3)(x+4) - 5(x-2) = (x-1)(x-4)$$

$$x^2 + x - 12 - 5x + 10 = x^2 - 5x + 4$$

$$-4x - 2 = -5x + 4$$

$$-4x + 5x = 4 + 2$$

$$x = 6$$

$$\boxed{x \in \{6\}}$$

b) $\frac{3x-1}{x-2} = \frac{5}{x-2} + 1$

Condition: $x \neq 2$

$$\frac{3x-1}{x-2} - \frac{5}{x-2} = 1$$

$$\frac{3x-1-5}{x-2} = 1$$

$$\frac{3x-6}{x-2} = 1 \Rightarrow$$

$$3x-6 = x-2$$

$$3x-x = -2+6$$

d) $\frac{1}{x-1} + \frac{1}{2} = \frac{2}{x^2-1}$

$$\frac{2(x+1)}{x-1} + \frac{x-1}{2} = \frac{2}{(x-1)(x+1)}$$

Condition: $\begin{cases} x \neq 1 \\ x \neq -1 \end{cases}$

$$LCO = 2(x-1)(x+1)$$

$$2(x+1) + x^2 - 1 = 4$$

$$2x + 2 + x^2 - 1 - 4 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x-1=0 \quad \text{OR} \quad x+3=0$$

$$x=1 \quad \quad \quad x=-3$$

not possible

$$x \in \{-3\}$$

$t-1=0$
 $t=1$ second
 The stone will be 256 ft above the ground after 1 second

e) $i = \frac{E}{R+r}$, $r = ?$

$$i(R+r) = E$$

$$R+r = \frac{E}{i}$$

$$r = \frac{E}{i} - R \quad \text{OR} \quad r = \frac{E - Ri}{i}$$

f) $m = \frac{Ry}{t}$, $t = ?$

$$mt = Ry$$

$$t = \frac{Ry}{m}$$

d) $t = ?$ if $h = 0$
 $0 = -16t^2 + 32t + 240$ / (-4)

$$16t^2 - 32t - 240 = 0$$

$$16(t^2 - 2t - 15) = 0$$

$$16(t-5)(t+3) = 0$$

$$t-5=0 \quad \text{OR} \quad t+3=0$$

$$t=5$$

$t = -3$
 not possible

So, the stone will hit the ground after 5 seconds

8) $h = -16t^2 + 32t + 240$

a) $t=0$, $h=240$ ft

b) $t=1$, $h = -16 + 32 + 240$
 $h = 256$ ft

c) $t = ?$ if $h = 256$

$$256 = -16t^2 + 32t + 240$$

$$16t^2 - 32t - 240 + 256 = 0$$

$$16t^2 - 32t + 16 = 0$$

$$16(t^2 - 2t + 1) = 0$$

$$16(t-1)^2 = 0$$

