

QUIZ #1 @ 85 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1) Solve the following equation by the square root property. Give exact answers.

$$(3x - 2)^2 = 40$$

2) Solve the following equations by the quadratic formula. Give exact answers.

a) $5 - \frac{2}{x} + \frac{1}{x^2} = 0$

b) $2x^2 - 3x + 1 = 0$

3) Solve the following equations:

a) $\frac{x+1}{2x} = \frac{3x-1}{5}$

b) $x(x+6) = 9$

c) $\sqrt{x+2} + 1 = \sqrt{2x+6}$

4) Solve the following inequality:

$$(x-1)(5-x)(x+2)^2 \leq 0$$

5) Factor the following expressions completely:

a) $4x^2 - 25$

b) $t^3 - 1$

c) $27 + y^3$

Solutions

$$\textcircled{1} \quad (3x-2)^2 = 40 \quad | \sqrt{}$$

$$\sqrt{(3x-2)^2} = \sqrt{40}$$

$$3x-2 = \pm\sqrt{40}$$

$$3x = 2 \pm 2\sqrt{10}$$

$$x = \frac{2 \pm 2\sqrt{10}}{3}$$

$$\textcircled{2} \quad \textcircled{a} \quad 5 - \frac{2}{x} + \frac{1}{x^2} = 0 \quad | \cdot x^2$$

$$\text{LCM} = x^2$$

Condition: $x \neq 0$

$$5x^2 - 2x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 4(5)(1)}}{2(5)} = \frac{2 \pm \sqrt{4 - 20}}{10}$$

$$x = \frac{2 \pm \sqrt{-16}}{10} = \frac{2 \pm 4i}{10}$$

$$x = \frac{2(1 \pm 2i)}{10} = \frac{1 \pm 2i}{5}$$

$$\boxed{x = \frac{1 \pm 2i}{5}}$$

$$\textcircled{b} \quad 2x^2 - 3x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{9 - 4(2)(1)}}{2(2)} = \frac{3 \pm 1}{4}$$

$$x = 1 \text{ or } x = \frac{1}{2} \quad \boxed{x \in \{1, \frac{1}{2}\}}$$

$$\textcircled{3} \quad \textcircled{a} \quad \frac{x+1}{2x} = \frac{3x-1}{5}$$

Condition: $x \neq 0$

$$5(x+1) = 2x(3x-1)$$

$$5x+5 = 6x^2 - 2x$$

$$6x^2 - 2x - 5x - 5 = 0$$

$$6x^2 - 7x - 5 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 4(6)(-5)}}{2(6)}$$

$$= \frac{7 \pm \sqrt{49 + 120}}{12} = \frac{7 \pm \sqrt{169}}{12}$$

$$= \frac{7 \pm 13}{12} \quad \leftarrow \quad \frac{7+13}{12} = \frac{20}{12} = \frac{5}{3}$$

$$\frac{7-13}{12} = \frac{-6}{12} = -\frac{1}{2}$$

$$\boxed{x \in \left\{ \frac{5}{3}, -\frac{1}{2} \right\}}$$

$$\textcircled{b} \quad x(x+6) = 9$$

$$x^2 + 6x - 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(1)(-9)}}{2}$$

$$= \frac{6 \pm \sqrt{72}}{2} = \frac{6 \pm 6\sqrt{2}}{2} = 3 \pm 3\sqrt{2}$$

$$\boxed{x = 3 \pm 3\sqrt{2}}$$

$$\textcircled{c} \quad \sqrt{x+2} + 1 = \sqrt{2x+6}$$

$$(\sqrt{x+2} + 1)^2 = (\sqrt{2x+6})^2$$

$$x+2 + 2\sqrt{x+2} + 1 = 2x+6$$

$$x+3 + 2\sqrt{x+2} = 2x+6$$

$$2\sqrt{x+2} = 2x+6 - x - 3$$

$$2\sqrt{x+2} = x+3$$

$$2\sqrt{x+2} = x+3 \quad |^2 \quad -2-$$

$$(2\sqrt{x+2})^2 = (x+3)^2$$

$$4(x+2) = (x+3)^2$$

$$4x+8 = x^2+6x+9$$

$$x^2+6x+9-4x-8=0$$

$$x^2+2x+1=0$$

$$(x+1)^2=0$$

$$x=-1$$

$$\text{check: } \sqrt{-1+2} + 1 = \sqrt{2(-1)+6}$$

$$1+1 = \sqrt{4} \quad \text{true}$$

$$\boxed{x \in \{-1\}}$$

$$(4) (x-1)(5-x)(x+2)^2 \leq 0$$

| | | | | | |
|-------------------|----|----|----|---|------|
| x | -∞ | -2 | 1 | 5 | ∞ |
| x-1 | - | - | 0+ | + | + |
| 5-x | + | + | + | + | 0--- |
| (x+2)^2 | + | + | 0+ | + | ++ |
| (x-1)(5-x)(x+2)^2 | - | 0- | 0+ | 0 | - |

$$x \in (-\infty, 1] \cup [5, \infty)$$

$$(5) (a) 4x^2-25 =$$

$$= (2x)^2 - 5^2$$

$$= (2x-5)(2x+5)$$

$$(b) t^3-1 = t^3-1^3$$

$$= (t-1)(t^2+t+1)$$

$$(c) 27+y^3 = 3^3+y^3$$

$$= (3+y)(3^2-3y+y^2)$$

$$= (3+y)(9-3y+y^2)$$