Math 150 Fall 07 Name:

QUIZ #3 @ 85 points

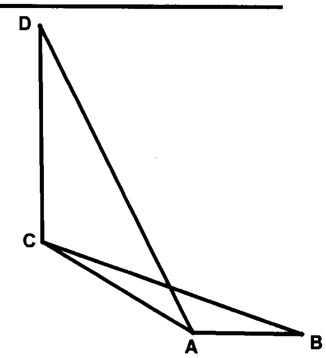
Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given.

1. Solve the triangle ABC knowing that $A = 50^\circ$, $B = 60^\circ$, and $a = 36km$.

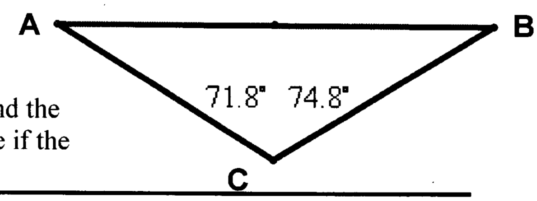
2. Solve the triangle ABC knowing that $A = 38^\circ$, $a = 41$ ft, and $b = 54$ ft.

3. Solve the triangle ABC knowing that $a = 48yd$, $b = 75yd$, and $c = 63yd$.

4. The figure shows how Colleen estimates the height of a tree that is on the other side of a stream. She stands at point A facing the tree CD and find that the angle of elevation from A to the top of the tree to be 51°. Then she turns 105° and walks 25 feet to point B, where she measures the angle between her path and the base of the tree. She finds that angle to be 44°. Find the height of the tree.



5. If you have ever ridden on a chair lift at a ski area and had it stop, you know that the chair will pull down on the cable, dropping you down to a lower height than when the chair is in motion. The figure shows a gondola that is stopped. Find the magnitude of the tension in the cable toward each end of the cable if the total weight of the gondola and its occupants is 1850 pounds.



6. a) Draw the vector $\vec{v}$ that goes from the origin to the point $(-2,3)$.
   b) Write the vector $\vec{v}$ in component form $\langle a,b \rangle$.
   c) Write the vector $\vec{v}$ in terms of the unit vectors $\vec{i}$ and $\vec{j}$.
   d) Find the magnitude of the vector.
   e) Find the angle θ, $0^\circ \le \theta < 360^\circ$ that the vector makes with the positive x-axis.

7. a) Find the dot product of the following two vectors: $\vec{u} = \vec{i} - 5\vec{j}$ and $\vec{v} = -\vec{2i} + 4\vec{j}$.
   b) Find the angle between the given two vectors.

8. Solve the following trigonometric equations:

   a) $2\sin^2 x - 3\sin x = -1$. Solve in $[0^\circ, 360^\circ)$.

   b) $2\sin\theta\cos\theta - \sqrt{3}\sin x = 0$. Find all real solutions (in radians).

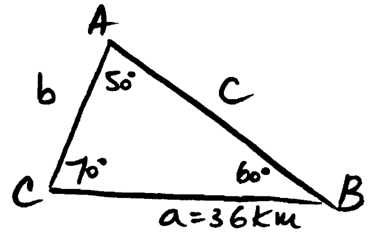
   c) $\cos 2x + 3\cos x - 2 = 0$. Solve in $[0^\circ, 360^\circ)$.

   d) $\sin 3x = \frac{1}{2}$. Find all real solutions (in radians).

SOLUTIONS

(1) $A = 50°$
$B = 60°$
$a = 36$ km

$C = ?$
$b = ?$
$c = ?$



Solution

If $A = 50°$, $B = 60°$, then
$C = 180° - 50° - 60°$
$\boxed{C = 70°}$

$b = ?$ $\quad \dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

$\dfrac{b}{\sin 60°} = \dfrac{36}{\sin 50°} \Rightarrow b = \dfrac{36 \sin 60°}{\sin 50°}$

$b \approx 40.7$ km
$\boxed{b \approx 41 \text{ km}}$

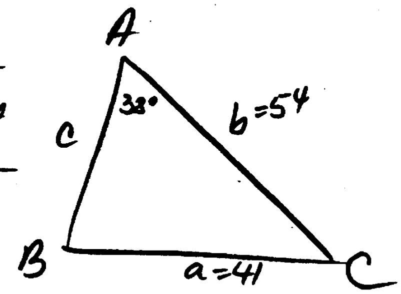
$c = ?$ $\quad \dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$\dfrac{c}{\sin 70°} = \dfrac{36}{\sin 50°} \Rightarrow c = \dfrac{36 \sin 70°}{\sin 50°}$

$c \approx 44.1$
$\boxed{c \approx 44 \text{ km}}$

(2) $A = 38°$
$a = 41$ ft
$b = 54$ ft

$B = ?$
$C = ?$
$c = ?$



Solution

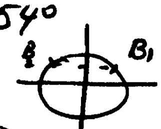
$B = ?$ $\quad \dfrac{\sin B}{b} = \dfrac{\sin A}{a}$

$\dfrac{\sin B}{54} = \dfrac{\sin 38°}{41} \Rightarrow$

$\sin B = \dfrac{54 \sin 38°}{41} \approx 0.81$

$\sin B \approx 0.81$

$B_1 = \sin^{-1}(0.81)$ OR $B_2 = 180° - B_1$
$B_1 \approx 54°$ $\quad B_2 \approx 180° - 54°$
$B_2 \approx 126°$



Case I

$A = 38°$
$\boxed{B_1 = 54°}$
$C = 180° - 38° - 54°$
$\boxed{C = 88°}$
$c^2 = b^2 + a^2 - 2ba \cos C$
$c^2 = 54^2 + 41^2 - 2(54)(41) \cos 88°$
$\boxed{c \approx 66.7 \text{ ft}}$

Case II

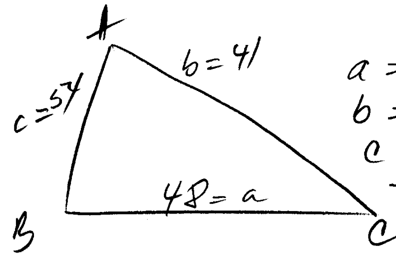
$A = 38°$
$\boxed{B_2 = 126°}$
$C = 180° - 38° - 126°$
$\boxed{C = 16°}$

$\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$\dfrac{c}{\sin 16°} = \dfrac{41}{\sin 38°}$

$\boxed{c \approx 18.4 \text{ ft}}$ $\quad c = \dfrac{41 \sin 16°}{\sin 38°} \approx 18.4$

(3)



$a = 48$
$b = 41$
$c = 54$

$A, B, C = ?$

Solution

$A = ?$ $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{75^2 + 63^2 - 48^2}{2(75)(63)} \approx 0.77$$

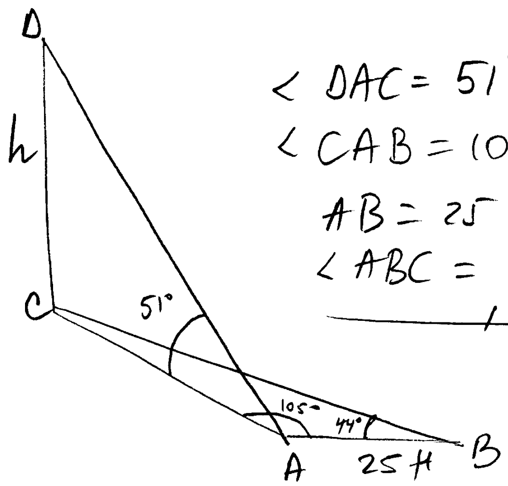
$$A = \cos^{-1}(0.77) \Rightarrow \boxed{A = 39.7^\circ}$$

$$\cos B = \frac{a^2 + b^2 - c^2}{2ab} \approx 0.11$$

$$B = \cos^{-1}(0.11) \Rightarrow \boxed{B = 83.7^\circ}$$

$$C = 180^\circ - A - B \Rightarrow \boxed{C \approx 56.6^\circ}$$

(4)



$\angle DAC = 51^\circ$
$\angle CAB = 105^\circ$
$AB = 25$ ft
$\angle ABC = 44^\circ$

$CD = ?$

Solution

$\triangle DCA$ - right at C

$$\tan 51^\circ = \frac{h}{AC} \Rightarrow h = AC \tan 51^\circ$$

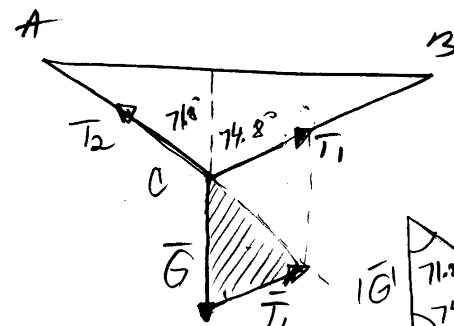
$\triangle ABC$: $\quad A = 105^\circ$
$B = 44^\circ$
so $C = 180^\circ - 105^\circ - 44^\circ$
$C = 31^\circ$

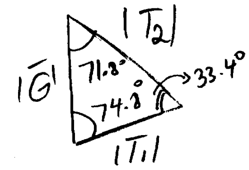
$$\frac{AC}{\sin B} = \frac{AB}{\sin C}$$

$$AC = \frac{25 \sin 44^\circ}{\sin 31^\circ}$$

$$\boxed{h = \frac{25 \sin 44^\circ}{\sin 31^\circ} \tan 51^\circ \approx 41.6\,\text{ft}}$$

(5)





$$\bar{G} + \bar{T}_1 + \bar{T}_2 = \bar{0}$$

$$|\bar{G}| = 1850 \text{ lbs}$$

$$\frac{|\bar{T}_1|}{\sin 71.8^\circ} = \frac{|\bar{G}|}{\sin 33.4^\circ}$$

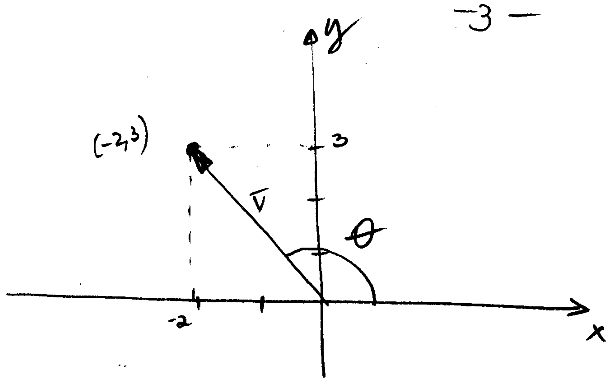
$$|\bar{T}_1| = \frac{1850 \sin 71.8^\circ}{\sin 33.4^\circ} \approx \boxed{3192 \text{ lbs}}$$

$$\frac{|\bar{T}_2|}{\sin 74.8^\circ} = \frac{|\bar{G}|}{\sin 33.4^\circ}$$

$$|\bar{T}_2| = \frac{1850 \sin 74.8^\circ}{\sin 33.4^\circ} \approx \boxed{3243 \text{ lbs}}$$

6

a)

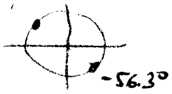


b) $\bar{v} = \langle -2,3 \rangle$

c) $\bar{v} = -2\bar{i} + 3\bar{j}$

d) $|\bar{v}| = \sqrt{2^2+3^2} = \sqrt{13}$

e) $\tan\theta = \frac{y}{x} = \frac{3}{-2}$



$\theta = \tan^{-1}(-1.5) \cancel{\approx -56.3°}$

or $\theta = -56.3° + 180°$

$\boxed{\theta \approx 123.7°}$

7 $\bar{u} = \bar{i} - 5\bar{j}$, $\bar{v} = -2\bar{i} + 4\bar{j}$

a) $\bar{u} \cdot \bar{v} = 1(-2) + (-5)(4) = -22$

b) $\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$

$|\bar{u}| = \sqrt{1^2+5^2} = \sqrt{26}$

$|\bar{v}| = \sqrt{2^2+4^2} = \sqrt{20}$

$\cos\theta = \frac{-22}{\sqrt{26}\sqrt{20}} \approx -0.9647$

$\theta = \cos^{-1}(-0.9647)$

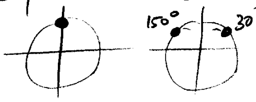
$\boxed{\theta \approx 164.7°}$

8 (a) $2\sin^2 x - 3\sin x = -1$

$2\sin^2 x - 3\sin x + 1 = 0$

$\sin x = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4}$

$\sin x = 1$ OR $\sin x = \frac{1}{2}$



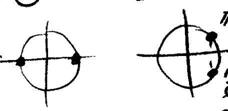
$x = 90°$ $x = 30°$ OR $150°$

$\boxed{x \in \{30°, 90°, 150°\}}$

(b) $2\sin\theta\cos\theta - \sqrt{3}\sin\theta = 0$

$\sin\theta(2\cos\theta - \sqrt{3}) = 0$

$\sin\theta = 0$ OR $2\cos\theta - \sqrt{3} = 0$



$\theta = k\pi$ $\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6} + 2k\pi$

or

$\theta = \frac{11\pi}{6} + 2k\pi$

$$\boxed{\begin{array}{l} \theta = k\pi \\ \text{or } \theta = \frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z} \\ \text{or } \theta = \frac{11\pi}{6} + 2k\pi \end{array}}$$

(c) $\cos 2x + 3\cos x - 2 = 0$

$2\cos^2 x - 1 + 3\cos x - 2 = 0$

$2\cos^2 x + 3\cos x - 3 = 0$

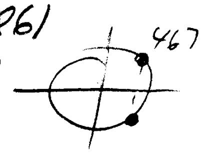
$\cos x = \frac{-3 \pm \sqrt{9 - 4(2)(-3)}}{2(2)}$

$\cos x = \frac{-3 \pm \sqrt{33}}{4}$

$\cos x \approx 0.6861$ OR $\cos x \approx -2.1 < -1$ not possible

$x = \cos^{-1} 0.6861$

$x \approx 46.7°$

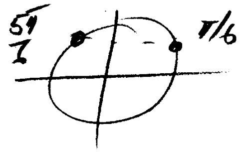


OR

$x = 360° - 46.7° = 313.3°$

$\boxed{x \in \{46.7°, 313.3°\}}$

(d) $\sin 3x = \frac{1}{2}$



$$\left\{\begin{array}{l} 3x = \frac{\pi}{6} + 2k\pi \\ \quad \text{OR} \\ 3x = \frac{5\pi}{6} + 2k\pi \end{array}\right. \Bigg/ \div 3$$

$$\boxed{\left\{\begin{array}{l} x = \frac{\pi}{18} + \frac{2k\pi}{3} \\ \quad \text{OR} \\ x = \frac{5\pi}{18} + \frac{2k\pi}{3} \end{array}\right.}$$